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ABSTRACT

The Virginia International Mathematics Assessment Project was undertaken in response to the need for high quality baseline data on student achievement and on curriculum and instructional issues. The assessment model chosen for this project used the design and instruments of the Second International Mathematics Study (SIMS). The study included 104 schools throughout Virginia, approximately 800 students and over 200 teachers and principals. Most students were in grades 8, 11, and 12. This report includes a summary of the findings with reference to achievement, opportunity to learn, attitude, and other background information on both teachers and students. Chapters include: (1) "Introduction"; (2) "Mathematics Achievement in Virginia Schools"; (3) "Opportunity to Learn Mathematics in Virginia Schools"; (4) "Student Attitudes"; (5) "Background Data"; (6) "Teachers Teaching...Students Learning"; (7) "Implementing SIMS at the Local Level"; (8) "Summary"; and (9) "Recommendations." Appended are a description of the five technical reports that accompany this study and a sample design. (KR)

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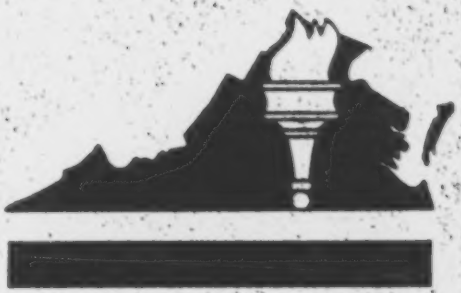
Virginia International Mathematics Assessment Project

MATHEMATICS COUNTS IN VIRGINIA



Main Report

Commonwealth of Virginia Department of Education
Mathematics Service; Richmond, Virginia 23216-2060

December 1989

This project was funded by the National Science Foundation. The views expressed therein do not necessarily reflect those of the National Science Foundation.

Virginia International Mathematics Assessment Project

MATHEMATICS COUNTS IN VIRGINIA

MAIN REPORT

COMMONWEALTH OF VIRGINIA DEPARTMENT OF EDUCATION
MATHEMATICS SERVICE; RICHMOND, VIRGINIA 23216
DECEMBER 1989

FORWARD

The Virginia International Mathematics Assessment Project was undertaken in response to the need for high quality baseline data on student achievement and on curriculum and instructional issues.

We live in an era of constant, rapid and dramatic changes, where tomorrow may bring about a complete alteration in the way that people live and work. The growing dependence of our society on high technology demands new types of education and training for our citizens. These changes will profoundly affect the mathematics curriculum and the way in which we teach mathematics. Recent national studies have indicated that much of the curriculum is outdated, overly abstract, and too theoretical.

To make intelligent decisions, however, about what changes are necessary, an assessment of mathematics education in our State was needed. Most of the assessment devices currently being used for student achievement and curriculum evaluation purposes were devised a number of years ago to serve quite different functions. We have chosen, instead, to use the design and instruments of the Second International Mathematics Study (SIMS). The SIMS model provides a rich context of data within which student outcomes can usefully be described and explained. We are grateful to the National Science Foundation for providing the necessary support to explore the value of the SIMS model as it is used in state educational systems.

The Study included 104 schools throughout Virginia, approximately 8000 students and over 200 teachers and principals. Most students in the Study were in grades 8, 11 and 12. A few students, however, were classified as tenth graders. This report includes a summary of the findings with reference to achievement, opportunity to learn, attitude and other background information on both teachers and students. There are five technical reports that accompany this report; copies of these reports are available upon request.

ACKNOWLEDGEMENTS

The Department of Education wishes to express appreciation to those who contributed to the success of this project. The study would not have been possible without the assistance of superintendents, central office personnel, principals, teachers, students, parents and the National Science Foundation which provided funding for the project.

Special appreciation is also expressed to the following people who made the project possible:

Dr. Kenneth Travers, Professor of Mathematics Education, University of Illinois, who served as a consultant and advisor in all aspects of the project and did much of the writing on the main report.

Dr. Edward Kifer, Jr., Associate Professor, University of Kentucky, who compiled, analyzed, and presented the data in a usable format.

Mrs. Sharon L. Bryant and Miss Patricia I. Wright, supervisors in the Mathematics Service of the Virginia Department of Education, for their participation in various aspects of the project.

Miss Joy Y. Hayes, senior secretary in the Mathematics Service of the Virginia Department of Education, who spent many hours typing the publication.

Dr. James E. Hecht, Director of Computing Services, Hinsdale, Illinois, for his assistance in planning and organizing the materials distribution.

Our thanks also go to each member of the Advisory Committee, the Curriculum Analysis Committee, and the Assistant Test Administrators for their strong support and assistance in this project, and to the University of Kentucky which permitted its computers to be used to compile the data.

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We must put Virginia in the first rank of states in the quality of education...not because of pride, but for the well-being of all Virginians—and yes, because of jobs.

Governor Gerald L. Baliles

What was the Virginia International Mathematics Assessment Project (VIMAP)?

The Virginia International Mathematics Assessment Project (VIMAP) used the methodology and findings of the Second International Mathematics Study (SIMS) to design and carry out a comprehensive evaluation of the eighth and twelfth grade mathematics programs in the Commonwealth of Virginia. The Project also calls for studying in detail the programs of two selected school divisions within the Commonwealth. The resulting information will be taken into account in curriculum reform activities that are now being planned.

This project was funded by the National Science Foundation with the expectation that it would provide other states, regions, and school divisions with a model that they could adapt to assist in their own assessment programs.

Rationale for VIMAP

There is widespread concern today with improving the quality of mathematics education at the school and college levels. Earlier in this decade, national reports such as the U. S. Department of Education's Nation at Risk and the National Science Board's Educating Americans for the Twenty-First Century provided the impetus for such concern. More recently, Everybody Counts and Science for All Americans offer guidelines for needed reform. Many states have followed suit with their own programs of upgrading education to meet the needs of an emerging high technology society.

Essential for all such reform efforts, however, is the need for reliable information on program quality at present. As the Council of Chief State School Officers has affirmed:

> "High quality data are essential to the continuing momentum of education reform. The publication of data brings education to the attention of the public and makes it an issue of enduring concern. By comparing the performance of school, district, or state with itself over time, or with other schools, districts, or states, data serve to exhort, motivate or reward."
>
> (page1, Position Paper Adopted by Council of Chief State School Officers, November, 1984.)

The Second International Mathematics Study offers an innovative approach to assessing the status of mathematics education at a state or district level. Furthermore, since the Second International Mathematics Study data are now archived, comparative information is available for the United States, including two provinces in Canada, several European countries and Japan. Use of the SIMS model of program assessment at the state level has also been made in California, Oregon, Iowa and Florida. In Canada it has been used at the provincial and school district levels.

Significance of VIMAP

In the Commonwealth of Virginia, several initiatives have been undertaken to reform mathematics education. In 1981, the Virginia State Department of Education initiated a program called the Standards of Learning (SOL). This program grew out of a Board of Education directive for improving education throughout the State. The program, which includes a set of objectives for mathematics, establishes a framework for instruction in the public schools. Subsequently, the Virginia Department of Education has undertaken the Mathematics/Technology Curriculum Project, designed to review the current mathematics offered in the state and to determine needed changes in content and emphasis as may be brought about by the changing nature of our technological society.

There is, however, no systematic way of evaluating the degree to which the goals of the mathematics curriculum are being reached. Virginia has a statewide testing program at three different levels where standardized achievement tests are used. The results from the achievement tests alone do not provide sufficient detailed information that is useful for making curriculum decisions on a statewide basis. The SIMS replication has helped provide the needed information for sustained curriculum reform efforts.

Relevance of VIMAP to Long Term Goals of the Commonwealth of Virginia

Critically important to formulating long-term goals of the State are reliable baseline data on certain facets of its mathematics program. The information included in the SIMS reports has already been useful in identifying major content areas and points of emphasis as indicated from the United States data. For example, the results from the VIMAP provide specific information needed as the State works toward its goals to train and retrain teachers, to teach a technologically-oriented curriculum, to provide local school divisions with instructional materials that are of high quality and reflect current thinking in mathematics, and to design programs that will assist our boys and girls in developing algorithmic thinking skills while de-emphasizing symbolic manipulation systems.

What was the Second International Mathematics Study (SIMS)?

The Second International Mathematics Study (SIMS) was a project of the International Association for the Evaluation of Educational Achievement (IEA) that involved a survey of the teaching and learning of mathematics in the schools of some 20 countries around the world. SIMS obtained data using national probability samples of students (classes) from each participating country. In particular, the achievement tests were internationally developed, based on a detailed analysis of the curriculum in each country.

Target Populations

In SIMS, two target populations were studied:

Population A: Students who were 13 years old by the middle of the school year (eighth grade in U.S.)

Population B: Students in the terminal grade of secondary school and who are continuing in their study of mathematics (in U.S., students in mathematics courses with prerequisite of at least algebra I, II and geometry).

For each of these target populations, random samples of classes (two per school) were drawn and the students were administered the tests and questionnaires. The mathematics teacher for each class also completed several questionnaires, including one that asked whether or not the mathematics required to respond correctly to each item on the student's test had been taught to the class. This information from the teachers, called "Opportunity to Learn Data", was very useful in helping to explain patterns of mathematics achievement that were found in SIMS (see, for example, the U. S. National Report on SIMS, The Underachieving Curriculum).

The SIMS Model

The Study is based on a model that views the curriculum from three points of view:

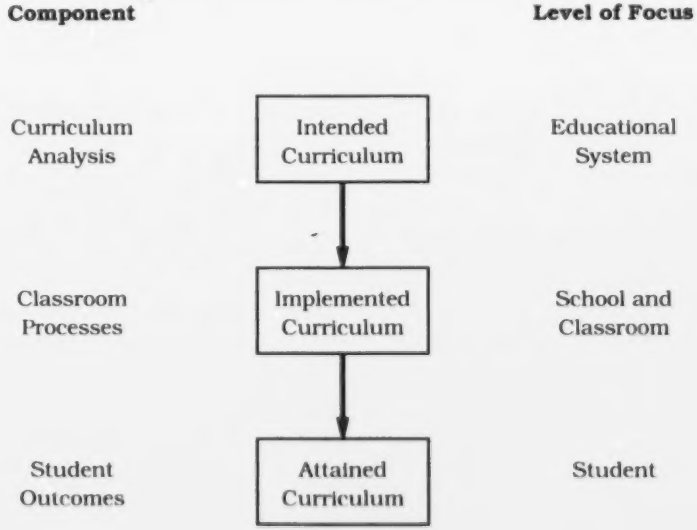
The intended curriculum: The mathematics that students in target populations are intended to know.

The implemented (taught) curriculum: The mathematics that the students were actually taught.

The attained curriculum: The mathematics that the students have attained or achieved.

This model provides a rich context of data that is very useful in explaining patterns of student achievement.

THREE ASPECTS OF A CURRICULUM



What were the major findings of SIMS as reported in The Underachieving Curriculum?

The Study provided detailed data on the teaching and learning of school mathematics in the United States from an international perspective.

Eighth grade

- U. S. achievement in arithmetic and algebra was at about the international average. But achievement in geometry and measurement was well below average, among the lowest one-fourth of the twenty participating countries.
- Teacher coverage of mathematics (called 'Opportunity-to-Learn') was highest in those countries having highest achievement. That is, those countries providing the greatest opportunity to learn mathematics were those in which student achievement was the greatest.
- In the United States there is not an eighth grade curriculum. The Study identified four curricula that had dramatically different mathematical content. This content ranges from grade school arithmetic for one of the class types (remedial) to high school freshman algebra for another. This differentiation between classes was more dramatic than in any country in the Study. In Japan, for example, all junior high school students are provided the same curriculum. In the seventh grade, in Japan, all students are taught algebra.
- From an international point of view, the U. S. curriculum has a great deal of repetition and is lean. Compared with many other countries, the curriculum lacks challenge and vitality. For example, it offers relatively little geometry, probability or topics in data analysis.

Twelfth Grade

- At this level, only students who are "mathematics majors" were tested. In the U. S. these were students having completed at least two years of algebra and one year of geometry. Nevertheless, U. S. student achievement on the international test was among the bottom one-fourth of all of the countries in the Study.
- The proportion of students in the United States that is studying advanced mathematics is about average for the countries in the Study. Several countries, including, for example, Canada (British Columbia) and Hungary, enrolled greater proportions of students in advanced mathematics classes. Japan enrolls about the same proportion of students as does the U. S.
- In most countries, all students who are in advanced mathematics study calculus. In the U. S., only about one-third of the advanced mathematics students enroll in calculus. (In Hungary, all students who remain in school– about 50% of the young people– are expected to study advanced mathematics, including calculus.)

How The Virginia Study was conducted

The Virginia International Mathematics Assessment Project was carried out in three phases:

Curriculum Analysis Phase

An analysis of the mathematics curriculum was conducted for the two target populations. This curricular information was used to assess the adequacy of the SIMS item pool for program evaluation in Virginia.

Testing Phase

Probability samples of classes were drawn from the entire state for each of the two target populations (see Appendix B for further details). The students were tested (Attitudes and Achievement) at the end of the 1986-87 school year. Data were obtained from teachers on "Opportunity-to-Learn", that is the extent to which the subject matter on the tests had been taught to the students being tested.

Analysis and Reporting Phase

The data were analyzed and preliminary reports generated for use by classroom teachers and supervisors in Virginia and personnel at the Virginia Department of Education. Reports were also prepared for national and international distribution.

Schedule of Activities

The schedule below indicates the main activities of the Virginia International Mathematics Assessment Project.

December 1984	Meeting with state supervisors
Summer 1985	NSF Proposal is funded
Fall 1985	Planning meeting for VIMAP
	Curriculum Analysis is begun
Spring 1986	Curriculum Analysis is concluded
Fall 1986	Sampling plan is completed
Spring 1987	Schools contacted for testing
Spring 1987	Orientation for teachers
	Data collected
Fall 1987	Data cleaned and organized for analysis
Spring 1988	Preliminary reports released
Fall 1988	Analysis continue
	Final report begun
Spring 1989	Final reports prepared
Fall 1989	Final reports released

Target Populations for Virginia

These populations were studied, to correspond with those in the international study:

Population A: All students enrolled in mathematics classes offered at eighth grade level.

Population B: All students enrolled in mathematics classes that have prerequisites of at least algebra I, geometry and algebra II.

Both Population A and Population B were important to the study in Virginia. Governor Gerald L. Baliles of Virginia appointed a Commission on Excellence in Education to make recommendations for moving Virginia into the top rank of states in the quality of education it offered. The Commission concluded that education in the middle school grades in Virginia was not living up to expectations nor keeping abreast of current trends in society. As a result of this report, the state Department of Education began an initiative to restructure the middle school. Since the eighth grade is the last year of middle school, it seemed appropriate that we assess the mathematics program and collect data on student achievement, attitudes of students and teachers, classroom processes, and the opportunity-to-learn mathematics. The SIMS model provides the framework to gather baseline data of this nature.

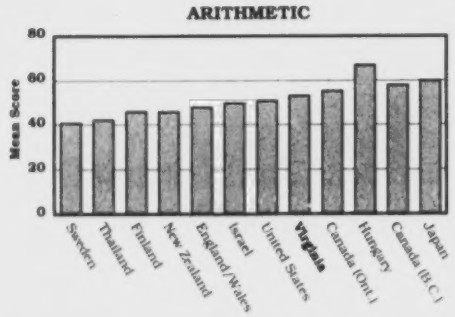
Several international studies on school mathematics have found that U. S. students at the twelfth grade level are not achieving at the level of students in some other countries. Comparison of student achievement in Virginia to that of the United States as a whole and with other countries make it possible to identify areas needing change, guidelines for curriculum changes and for inservice and preservice education.

Numeracy is the ability to cope confidently with the mathematical demands of adult life.

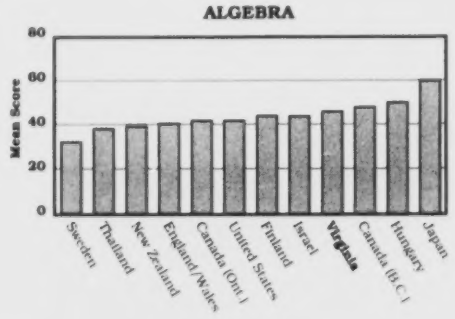
Mathematics Counts

(Inquiry into the teaching of mathematics in England and Wales, 1982).

Eighth Grade

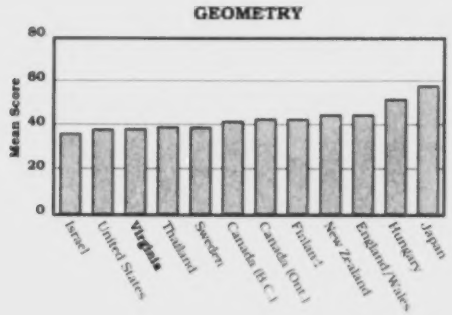


Eighth grade achievement in arithmetic is about at the international average for the United States and a few points above the U.S. for Virginia.

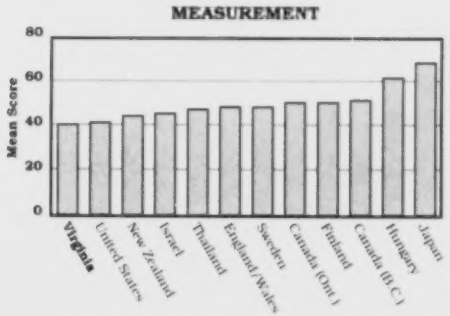


Eighth grade scores in algebra were, on average, somewhat higher in Virginia than in the United States as a whole.

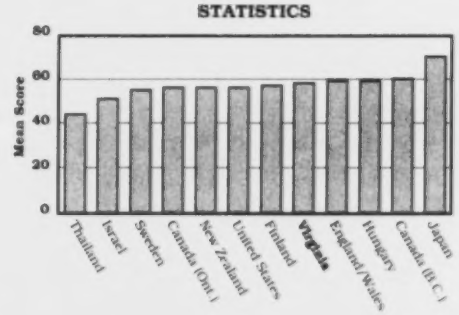
Eighth Grade



In geometry, Virginia scored just slightly above the United States as a whole, which was among the bottom one-fourth of the countries in the Study. An important goal of the international investigation was to find reasons for this performance.



In measurement, eighth grade students in Virginia, scored slightly below the United States as a whole, which was among the lowest one-fourth of the countries. It is important to note that, since the test was internationally developed, the metric system of units was used (the metric system is <u>the</u> system of measurement in all other countries in the study.) However, not all of the items required a knowledge of the metric system itself. Rather, they require understanding concepts of measurement and how to apply them.



Statistics achievement, a measure of students' ability to understand and work with data, was a little higher in Virginia than in the United States as a whole. In many countries, including the U. S., this aspect of the curriculum is receiving increased attention in current reform efforts.

The overall pattern of Virginia scores corresponds remarkably with that of the U. S. scores. For example, in the topic areas of arithmetic and algebra, the U. S. was at the international mean. Virginia scored a few points above the U. S. on these topics. In geometry and measurement, the U. S. was somewhat below the international mean. The scores on these two topics for Virginia are virtually identical to those of the U. S. In statistics, the U. S. scored a few points above the international mean and Virginia scored four points above the international mean.

Achievement Comparisons:
Virginia vs. the U. S. and International
Subtest Scores: Eighth Grade
(Mean Percent Correct)

Topic	Virginia				United States		International
	Overall	Remedial	General	Pre-Algebra	Algebra	Mean	Mean
Arithmetic	53	30	44	58	76	51	51
Algebra	46	23	35	50	75	43	43
Geometry	38	25	33	40	55	38	41
Statistics	59	41	51	64	78	57	55
Measurement	41	27	35	44	59	42	51

What conclusions might be drawn from these findings? We first must remind ourselves that, overall, the U. S. scores at this grade level are close to the international mean which suggests that achievement in some other countries was higher. While a particular score may not be important, it is important that we look at our state goals for eighth grade mathematics and decide where they are, and where emphasis should be placed. For instance, in Virginia we have a goal to provide an opportunity for all students to study algebra, including the lower level classes. The findings of this study clearly support the implementation of this goal. It should be noted, however, that Virginia students enrolled in algebra classes scored significantly higher than the U. S.

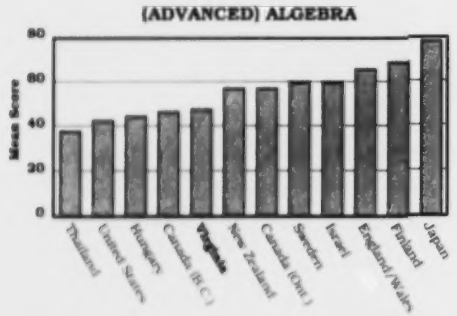
The VIMAP data indicate very clearly, however, that considerable attention needs to be paid to the topics of measurement and geometry. While some of the reasons for our low performance on measurement may be attributed to the fact that only the metric system was included on the test, not all of the blame can be placed there. For example, only about one-half of our students are able to read a scale (admittedly, only 40% of the students across the countries could do so--but that does not detract from the basic importance of this topic!).

With respect to statistics, at first glance it may appear that all is well, with Virginia scoring four points above the international mean! But we are not sure, we must look at our curriculum to see, if indeed, we are where we would like to be in this important area of mathematics. Here in Virginia, as in many other places, great importance is attached to the ability to work with data--to collect and organize numerical information in such a way that it can tell a story effectively. The VIMAP data suggest that we may be in reasonable shape with routine tasks, such as reading a bar graph, but have we reached our goal of quantitative literacy for all of our youngsters in school?

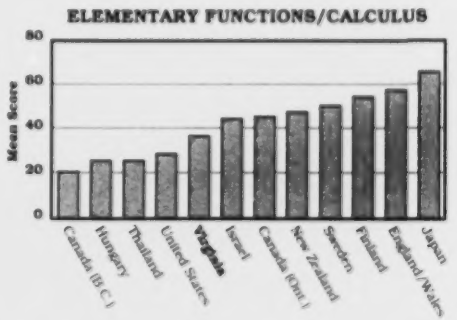
The national spotlight is turning on mathematics as we appreciate its central role in the economic growth of this country...it must become a pump instead of a filter in the pipeline.

R. M. White (as cited in Everybody Counts, 1989)

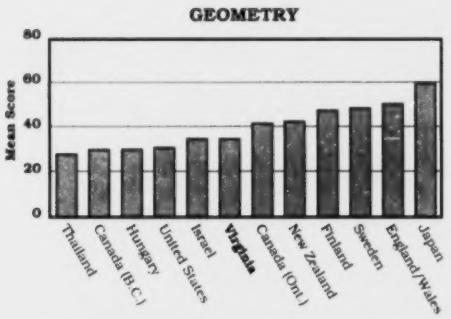
Twelfth Grade



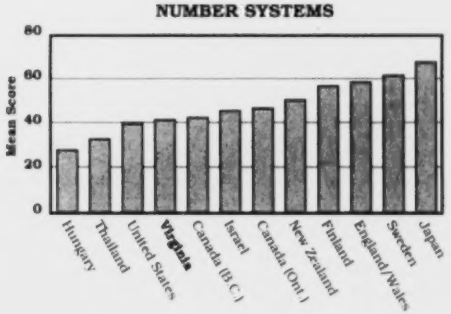
Advanced algebra and calculus provide the basis for competence in science and technology. The data for the twelfth grade represent the achievement of our most advanced mathematics students--those who are enrolled in mathematics courses requiring at least algebra I, geometry and algebra II. Virginia students scored, on average, higher than the United States as a whole, in both advanced algebra and calculus.



Twelfth Grade



Geometry and number systems, show patterns of achievement similar to algebra and calculus. That is, the U. S. is among the lowest one-fourth of the countries and Virginia tends to be a few points higher in achievement than the U. S.



Virginia and the United States
Subtest Scores: Twelfth Grade
(Mean Percent Correct)

Topic	Virginia			United States			International
	Pre-Calc	Calc	Over-all	Pre-Calc	Calc	Over-all	Mean
Sets & Relations	45	60	49	54	64	56	61
Number Systems	41	50	42	38	48	40	47
Algebra	44	61	48	40	57	43	57
Geometry	33	41	35	30	38	31	42
Functions/Calculus	30	57	37	25	49	29	46
Probability/Statistics	38	55	41	39	48	40	46

Again, as with the eighth grade, there is a correspondence between the scores of the Commonwealth and of the entire U. S. This is particularly so for the pre-calculus classes. For the calculus classes, Virginia tends to score a few points above those of the U. S. On the calculus subtest, the difference is dramatic, with Virginia students eight points above the U. S. -- and among the top one-fourth of all countries.

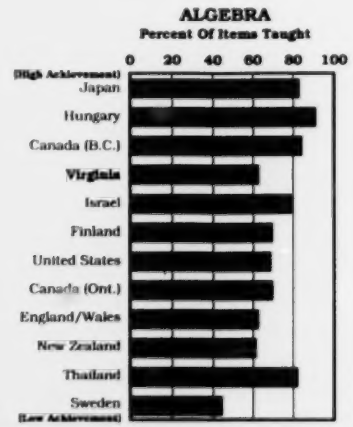
The topic area where Virginia's performance was lowest was sets and relations. Further analysis of the data will be done to determine reasons for this. It is likely, for example, that the topics are not covered heavily by many Virginia teachers. In geometry, Virginia students obtained an average score above the U. S. but a few percentage points below the international mean.

Today's mathematics opens doors to tomorrow's jobs. As successive waves of immigrants have used this country's educational system to secure better lives for themselves and their children, so today's children the world over are using mathematical training as a platform on which to build up their lives. America's children deserve the same chance.

Everybody Counts 1989

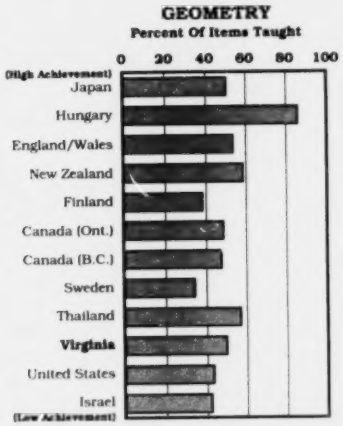
The Second International Mathematics Study provided a new kind of data—data that were not available in any other kind of mathematics survey at that time. The information provided, called "Opportunity-to-Learn", was obtained by asking teachers to indicate, for each item on the international test, whether or not that mathematics had been taught to the class, either during the school year, or prior to that school year. The same kind of data was gathered in the Virginia study.

Eighth Grade

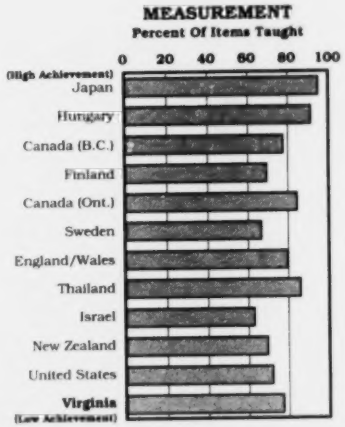


In this graph, international and Virginia achievement data are shown for eighth grade algebra (seventh grade in Japan). Virginia, along with the countries are ranked from high to low in achievement. The length of the bar for each country shows the amount of teacher coverage of the algebra items on the international test. (Hungarian teachers reported highest coverage.) Generally, the graph suggests, the higher-achieving countries were those having higher teacher coverage.

Eighth Grade



Opportunity-to-Learn geometry for eighth grade students was, on average, relatively low for students in all countries. The graph indicates that typically less than one-half of the geometry on the international test was covered by the teachers (note the exception for Hungary!). Virginia teachers reported covering a little more geometry than U. S. teachers as a whole--and student achievement was somewhat higher.



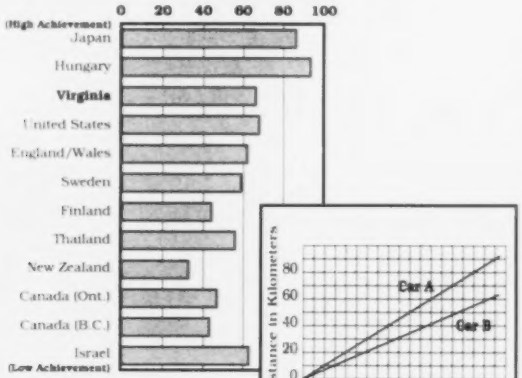
Teacher coverage of the topics on measurement was reported to be generally high in all countries and Virginia. Even in the United States, where the metric system is not yet commonly used, coverage was around seventy percent. The low student achievement in the United States generally, and in Virginia in particular, warrants closer scrutiny in the light of the high teacher coverage of the measurement items.

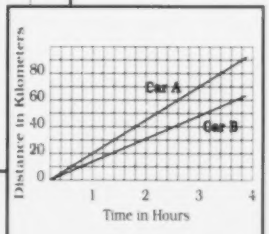
Eighth Grade

The following selected items help to illustrate the findings of the Virginia International Mathematics Assessment Project. For each item, reported teacher coverage and associated student achievement are shown.

STATISTICS

Percent Of Items Taught





Three hours after starting, car A is how many kilometers ahead of car B?

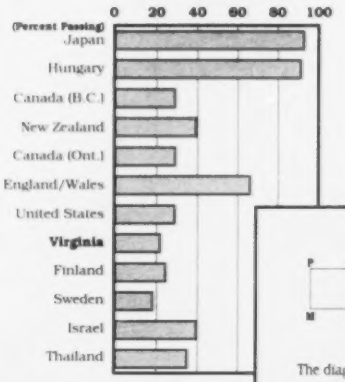
A 2 C 15 E 25
B 10 *D 20

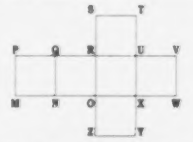
160

40

GEOMETRY

Percent Of Classes Taught



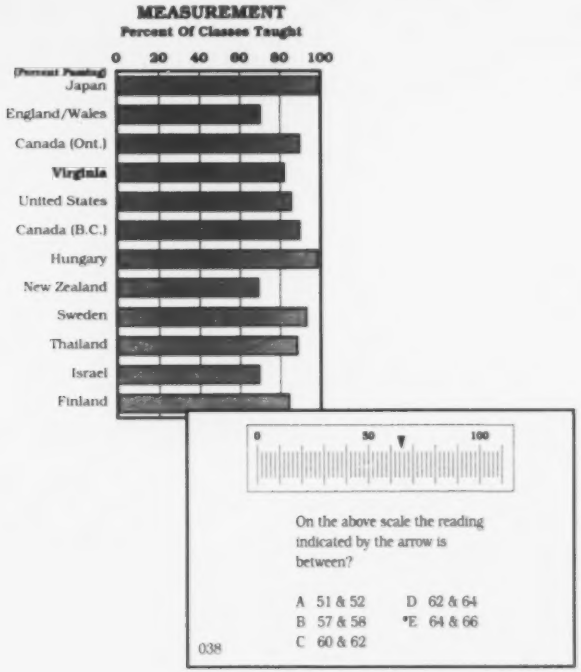


The diagram shows a cardboard cube which has been cut along some edges and folded out flat. If it is folded to again make the cube, which two corners will touch corner P?

A Q & S *D T & V
B T & Y E U & Y
C W & Y

031

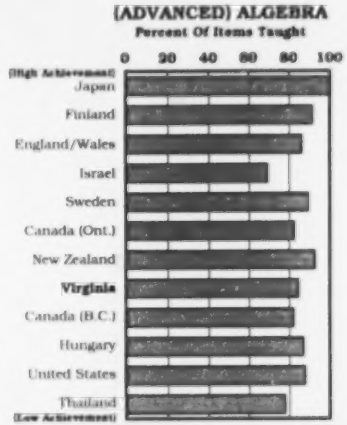
41



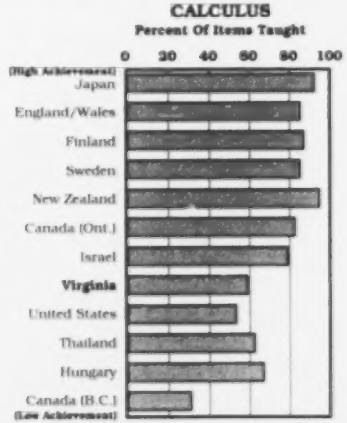
Since mathematics is the foundation of science and technology, it serves as a key to opportunity and careers.

Everybody Counts (1989)

Twelfth Grade

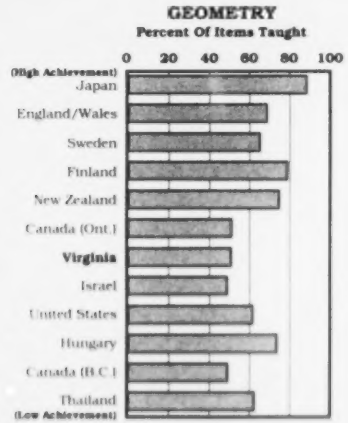


Teachers report high coverage of the advanced algebra topics in all countries and in Virginia. The low student achievement in some of these countries is, apparently, due to factors other than low teacher coverage.

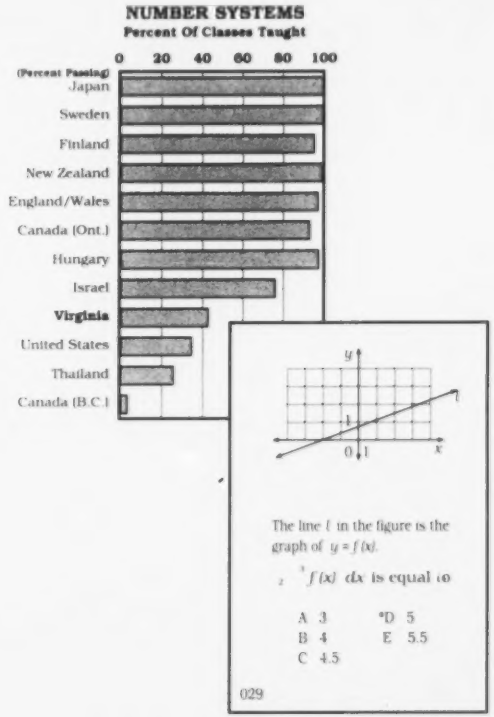


In most countries in the international study, calculus is a standard topic in advanced mathematics classes (twelfth grade in the U. S.) However, in the United States including Virginia, only a relatively small proportion of advanced mathematics students take calculus.

Twelfth Grade



There is little agreement internationally as to what topics should be included in geometry. The relatively low teacher coverage for geometry reflects this lack of consensus.



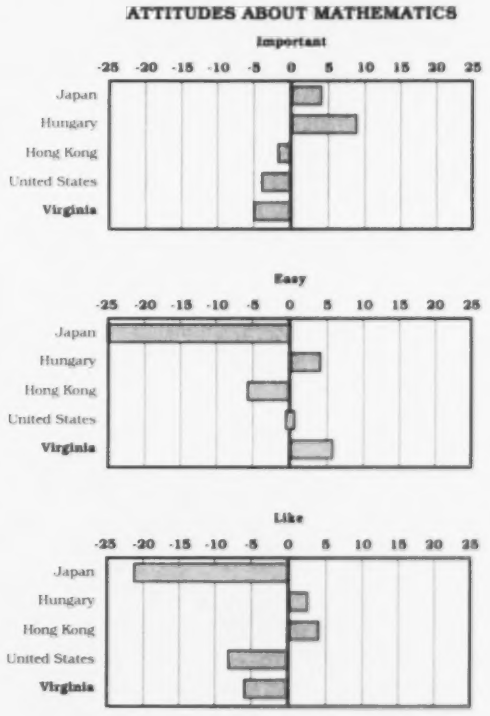
In the U. S., and in Virginia, about one out of three advanced mathematics students was taught this topic.

...We see classrooms as places where interesting problems are regularly explored using important mathematical ideas.

NCTM Curriculum and Evaluation Standards for School Mathematics (1989)

Attitudes of Students

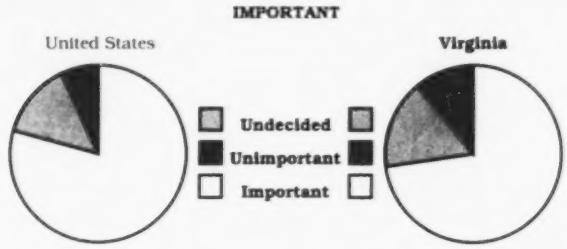
Gaining knowledge about mathematics is not the only important outcome of education. How students feel about mathematics, and how they view the subject are also of great concern.

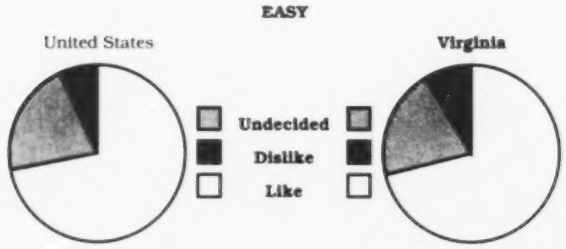


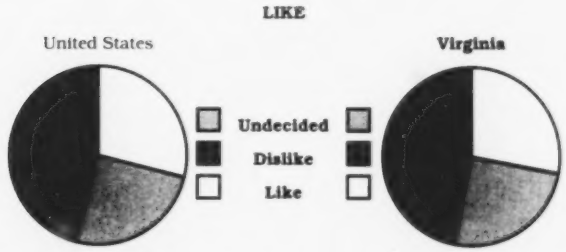
The graphs here report data on how students regard activities in mathematics classrooms with respect to their importance, ease and enjoyment.

Checking Work
Eighth Grade

Checking your work in mathematics class was regarded as important by the vast majority of eighth grade students in Virginia--as it was by their counterparts across the United States. Checking work was also seen as being rather easy, but students tended to not like doing it. Nearly one-half of the students in both Virginia and the U. S. as a whole reported disliking checking their work.

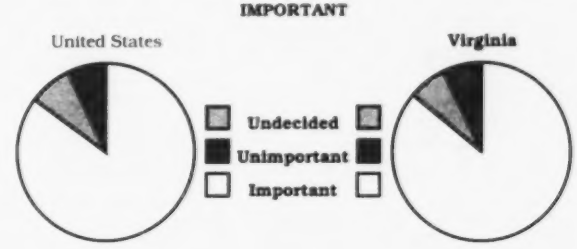


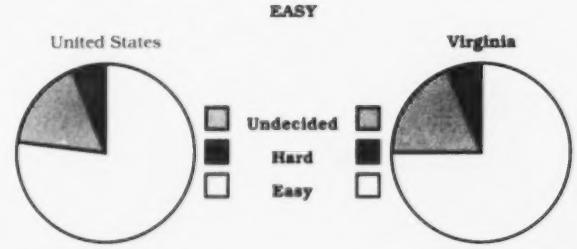


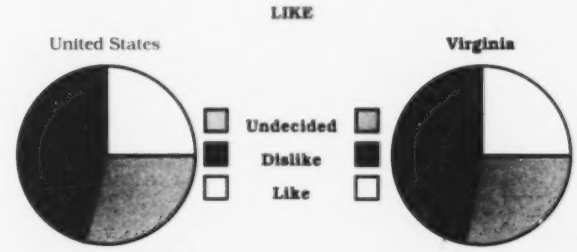


Twelfth Grade

The attitudes of twelfth grade mathematics students toward checking their work are remarkably similar to those of the eighth graders. They tend to feel that it is important to do, is easy, but they do not like to do it.

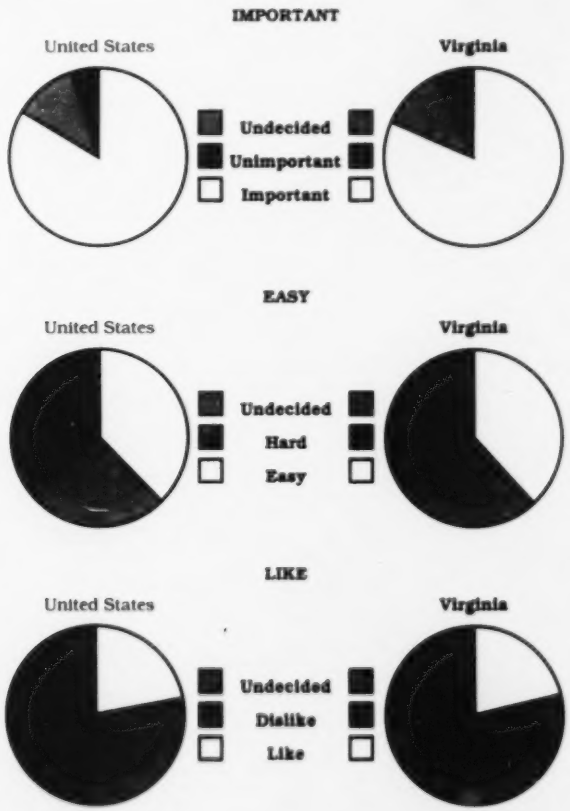






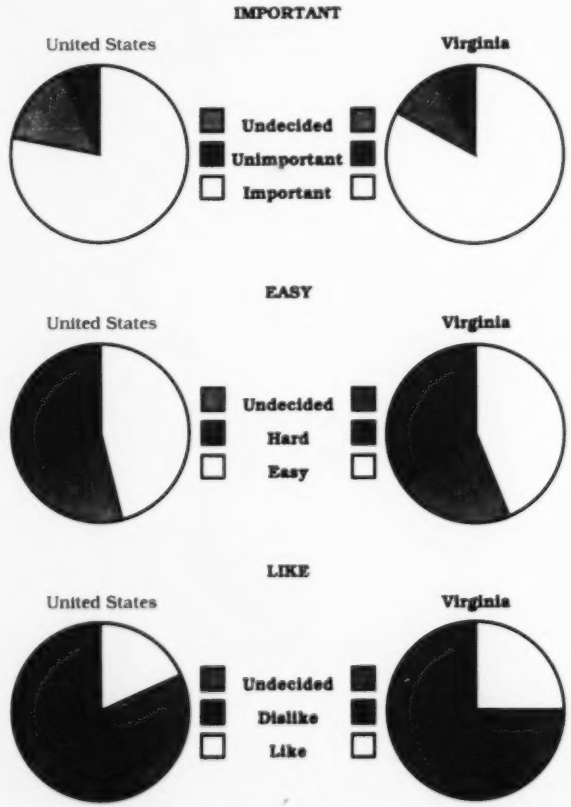
Memorizing Rules and Formulas
Eighth Grade

Mathematics as mostly memorizing seems to be a popular view held by many Virginia eighth graders, and by almost exactly the same proportion of U. S. eighth grade student, too. The students (in both Virginia and the entire country), by a vast majority, regard it as important to memorize rules and formulas, but less than one-fourth of the students like to memorize. (Note the contrast of these attitudes with their attitudes toward using calculators.)



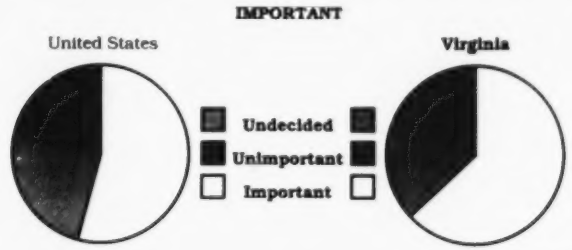
Twelfth Grade

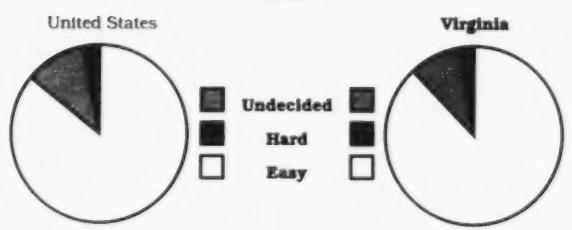
Mathematics as mostly memorizing was an attitude about mathematics held by Virginia's advanced mathematics students (and by those across the U. S.). About 80% of the advanced students regard it as important to memorize, nearly one-half of them dislike doing so. Interestingly, nearly one-half of the advanced students report it easy to memorize rules and formulas.



Using Calculators
Eighth Grade

Using calculators is liked by nearly 80% of the Virginia eighth graders and by three-fourths of the U. S. students at this level. Nearly 90% of the Virginia students state that it is easy to use calculators. However, the students at this grade level are less strong in their feelings about the importance of calculators.

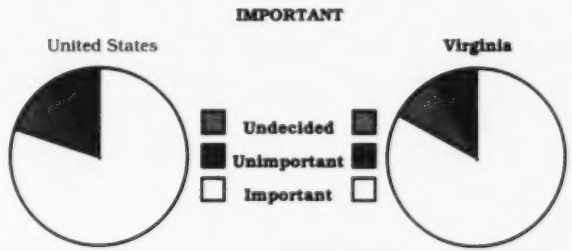


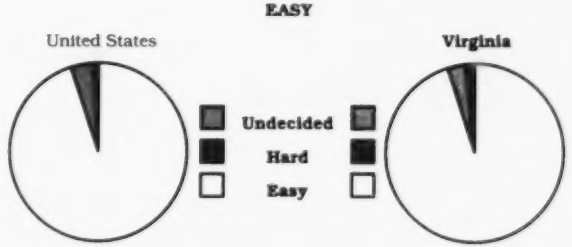


52

Twelfth Grade

Using calculators in advanced mathematics is regarded as easy by nearly all of the students and is liked by 85% of them. They are also in rather strong agreement about the importance of calculators in advanced mathematics.

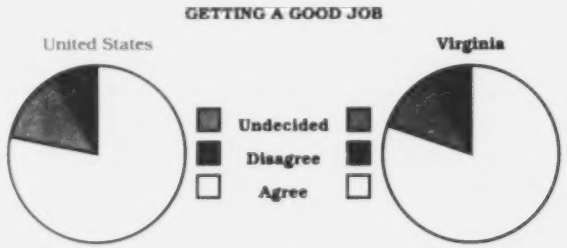




53

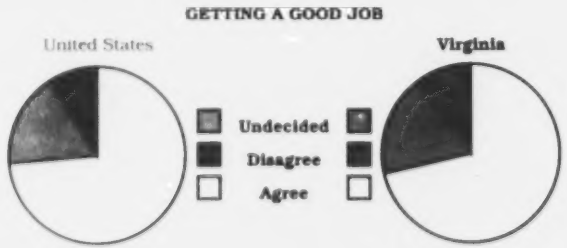
Mathematics and Society
Eighth Grade

It is important to know mathematics in order to get a good job. Knowing mathematics will help to get a good job is an attitude shared by the vast majority of Virginia eighth graders. In the light of this attitude, one wonders the extent to which students are motivated to continue in the study of mathematics throughout high school.



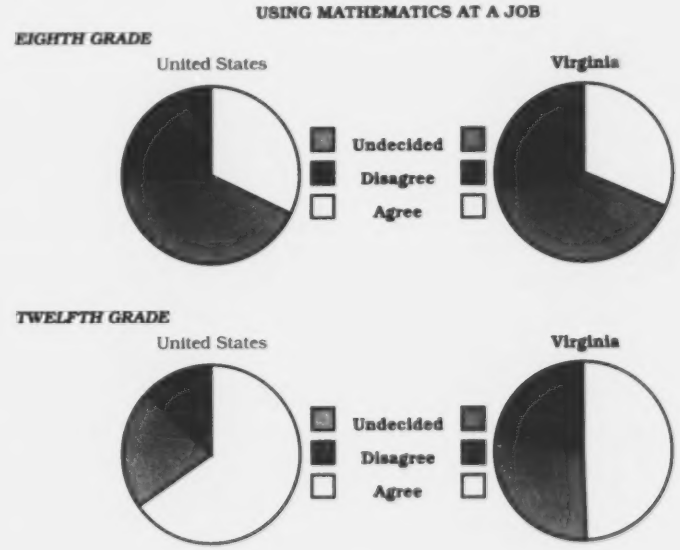
Twelfth Grade

The role of mathematics in helping one get a good job is regarded as important by twelfth graders in Virginia.



54

I would like to work at a job that lets me use mathematics. Wanting to use mathematics on the job was an attitude of only about one-third of the eighth grade students. However, at the twelfth grade level, about one-half of the advanced mathematics students look to using mathematics in employment.



55

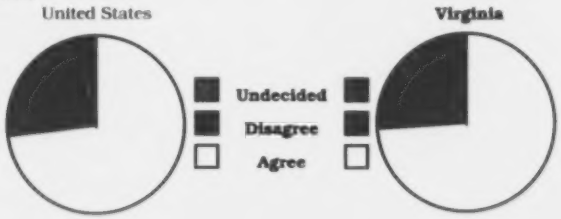
Mathematics and Society
Eighth and Twelfth Grade

Mathematics is useful in solving everyday problems. Eighth grade students report a belief that mathematics is useful in solving everyday problems. This would appear to provide an opportunity to capitalize on this interest and incorporate more of such problems in the curriculum.

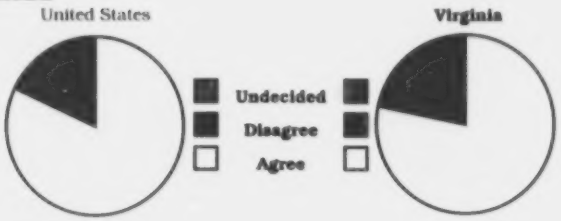
Advanced mathematics students are even more in agreement about the importance of mathematics in the real world. Again, an apparently important opportunity for curriculum and instruction!

SOLVING EVERYDAY PROBLEMS

EIGHTH GRADE



TWELFTH GRADE

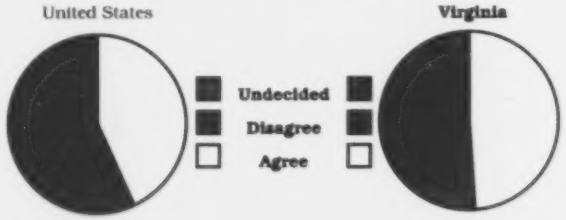


Word problems are more fun when you use a calculator. Calculators can help make mathematics class more enjoyable. Many eighth grade students seem to believe this. Perhaps this provides encouragement for making greater use of this teaching device.

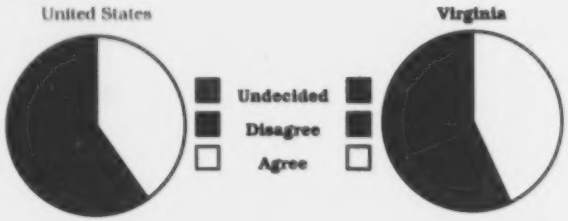
Advanced mathematics students tend to like to have calculators available when doing word problems, too. Apparently, calculators are regarded by many students as a tool that adds enjoyment to class.

WORD PROBLEMS ARE MORE FUN WITH A CALCULATOR

EIGHTH GRADE



TWELFTH GRADE



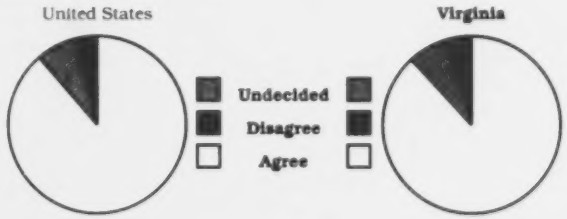
Mathematics and Myself
Eighth and Twelfth Grade

I really want to do well in mathematics. The desire of most (nearly 90%) eighth grade students in Virginia to do well in mathematics would seem to be a source of encouragement to mathematics teachers, administrators and parents.

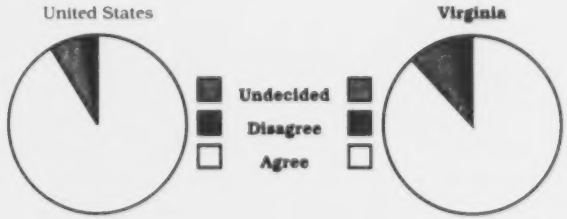
Advanced mathematics students are virtually unanimous in their desire to do well in mathematics. These students have remained in mathematics classes for most if not all of their high school experience.

I REALLY WANT TO DO WELL IN MATHEMATICS

EIGHTH GRADE



TWELFTH GRADE



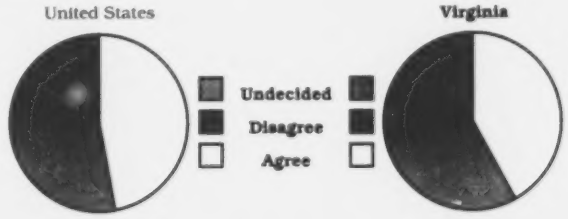
58

I am looking forward to taking more mathematics. Eighth grade students are less in agreement about the prospect of taking more mathematics--in spite of their recognition of its importance in getting a good job.

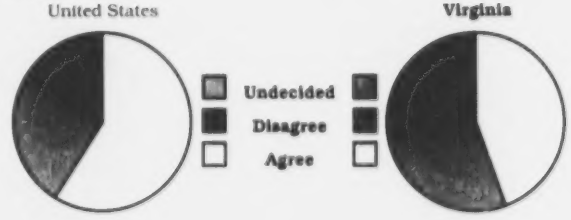
Twelfth grade students in advanced mathematics class in Virginia are less in agreement in their looking forward to more mathematics than are their U. S. counterparts. Less than one-half of the Virginia students reported they were looking forward to more mathematics while nearly sixty percent of the U. S. students were.

I AM LOOKING FORWARD TO TAKING MORE MATHEMATICS

EIGHTH GRADE



TWELFTH GRADE



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What Mathematics Is Like
Eighth and Twelfth Grade

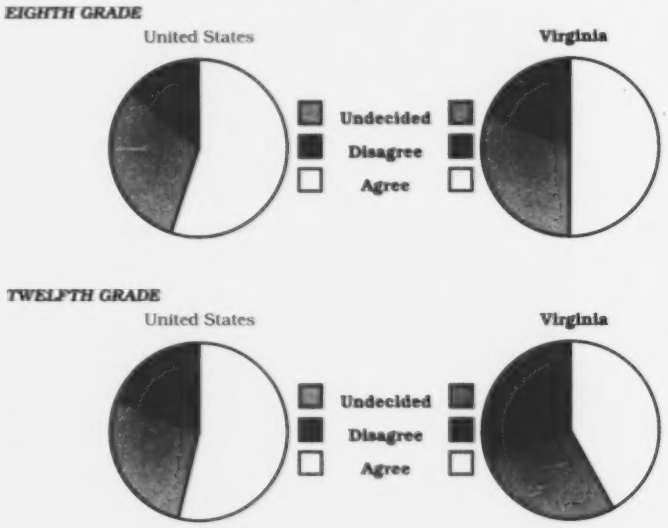
Students learn the subject matter of mathematics (arithmetic, algebra, and so on) but they also learn things about the nature of mathematics--what is mathematics like? Is it, for example, a fixed body of knowledge that has remained much the same for the past 100 years or so? Does mathematics consist of sets of rules and formulas that are to be memorized and given back on tests? Does doing well in mathematics mainly mean having a good memory?

In the Second International Mathematics Study a set of items was devised in order to measure how one views mathematics as a field of study.

Mathematics is a good field for creative people. Mathematics as a good field for creative people was a point of view held by approximately one-half of the Virginia eighth graders and by only slightly more in the U. S. sample. This seems to indicate that students see little room for creativity in mathematics.

Mathematics as a good field for creative people is regarded by an even smaller proportion of advanced mathematics students than the eighth grade group. Less than one-half of the Virginia students reported seeing mathematics in this light.

MATHEMATICS IS A GOOD FIELD FOR CREATIVE PEOPLE



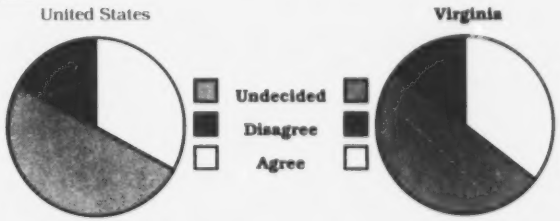
What Is Mathematics Like
Eighth and Twelfth Grade

New discoveries in mathematics are constantly being made. Only about one-third of the eighth grade students report believing that mathematics is a changing field.

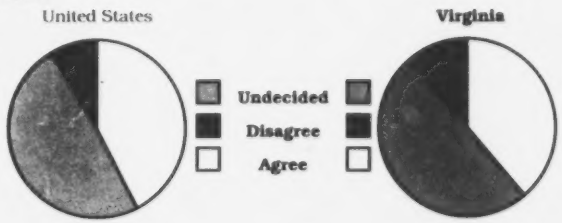
Close to the same proportion of advanced mathematics students as eighth graders regard mathematics as having new discoveries. This view of mathematics as a fixed body of knowledge is a disconcerting one, especially when found in those at the end of their secondary school studies.

NEW DISCOVERIES IN MATHEMATICS ARE CONSTANTLY BEING MADE

EIGHTH GRADE



TWELFTH GRADE



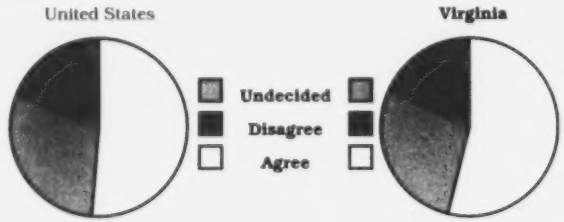
62

A mathematical problem can always be solved in many different ways. In Virginia, over one-half of the students in the eighth grade see that there can be flexibility in solving mathematics problems. Fortunately, this attitude has somewhat increased for those students in the twelfth grade advanced mathematics classes, and they realize that there can be multiple solutions to mathematics problems.

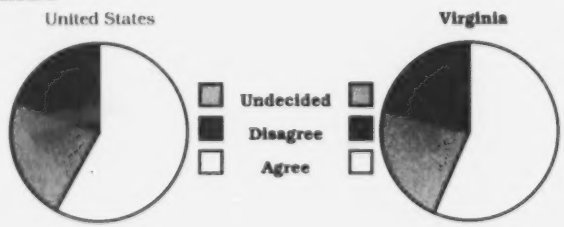
In both of the grade levels for both the U. S. and Virginia, about one-fourth of the students disagree and one-fourth are undecided about whether a problem can be solved in many different ways. This means that we have a long way to go in order to dispel the attitude that there is only one solution when solving a mathematical problem.

A MATHEMATICS PROBLEM CAN ALWAYS BE SOLVED IN DIFFERENT WAYS

EIGHTH GRADE



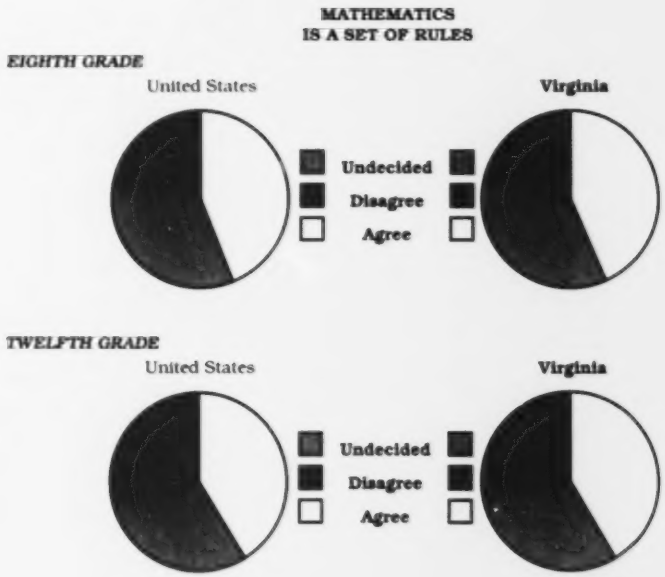
TWELFTH GRADE



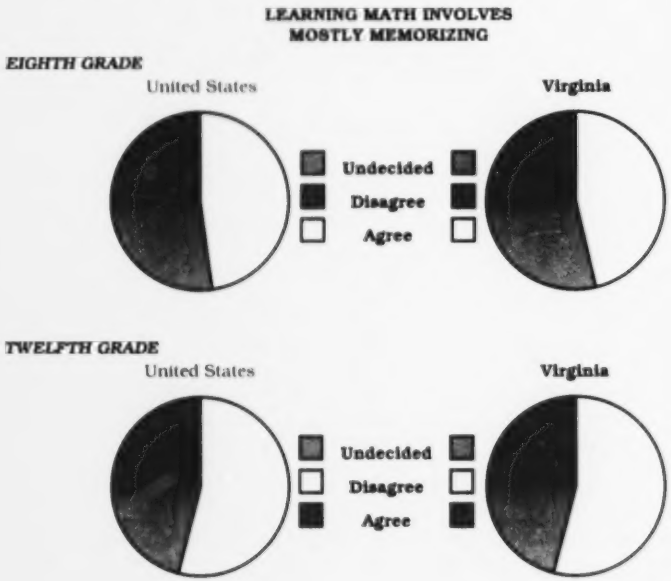
63

What Is Mathematics Like
Eighth and Twelfth Grade

Mathematics is a set of rules was an opinion endorsed by over forty percent of the eighth grade students in Virginia. Distressingly, about the same proportion of twelfth grade students shared this opinion of mathematics.



Nearly one-half of the eighth graders also expressed agreement with the idea that learning mathematics involves mostly memorizing. On the other hand, the eighth grade students did have a somewhat open-ended view of mathematics in that nearly one-half of them agreed that a mathematics problem can always be solved in different ways. Nearly sixty percent of them agreed that estimating is an important mathematical skill. Again, the opinions of the Virginia students corresponding very closely with those of the U. S. students.

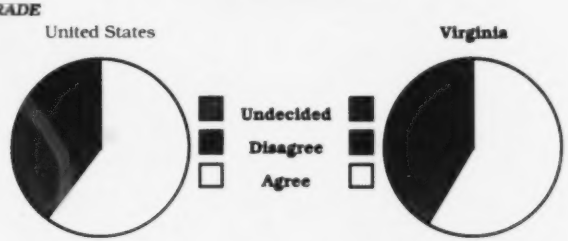


What Is Mathematics Like
Eighth and Twelfth Grade

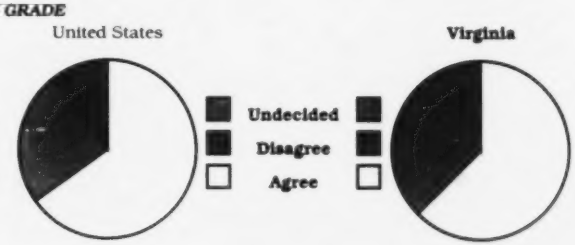
The attitudes of the advanced mathematics students toward mathematics as a subject of study were similar to those of the eighth grade students in that over forty percent view mathematics as a set of rules and over one-half agree that a mathematics problem can be solved in different ways. However, evidence of some maturity in mathematical thinking could be inferred from the finding that over fifty percent of the advanced students disagreed with the statement that learning mathematics involves mostly memorizing.

ESTIMATING IS AN IMPORTANT MATHEMATICAL SKILL

EIGHTH GRADE



TWELFTH GRADE



66

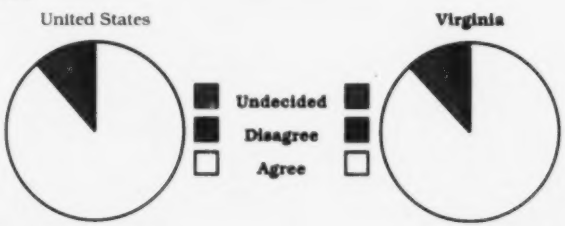
Home Support
Eighth and Twelfth Grade

Home support. One of the best documented findings in educational research is that support from the home is a very powerful predictor of a student's success in school. The following items were designed to measure the extent of support and encouragement from the home for studying mathematics.

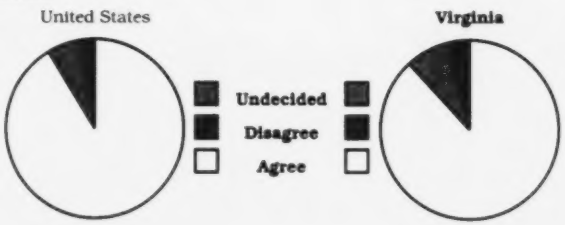
My parents want me to do well in mathematics class. Nearly all of the students in the eighth grade classes agreed with this statement. At the twelfth grade level, approximately 90% of the students agreed with the statement.

I REALLY WANT TO DO WELL IN MATHEMATICS

EIGHTH GRADE



TWELFTH GRADE



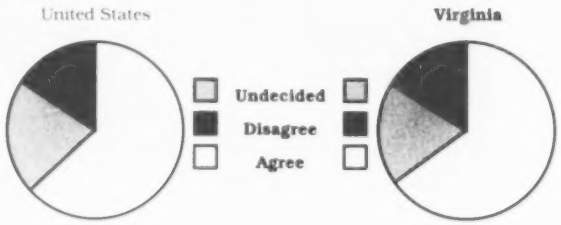
67

Home Support
Eighth and Twelfth Grade

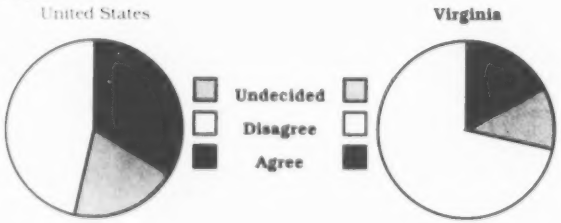
My parents are interested in helping me do mathematics. At the eighth grade level, about two-thirds of the students agreed with this statement. However, at the twelfth grade level, far fewer students held this view. Only about one in five of these students see their parents as interested in helping them with their mathematics. This could be that the students regard their parents as not having the needed background to be helpful at this level of mathematics.

MY PARENTS ARE INTERESTED IN HELPING MY DO MATH

EIGHTH GRADE



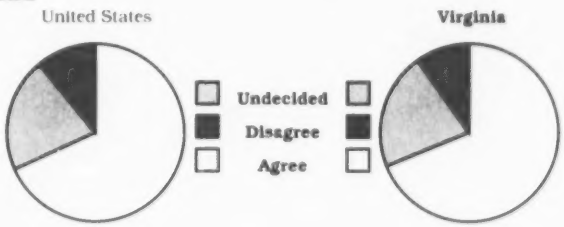
TWELFTH GRADE



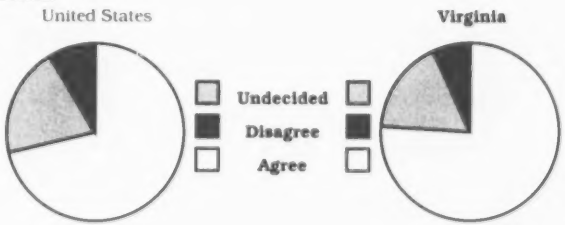
Boys need more mathematics than girls. About two-thirds of the eighth grade students disagreed with the opinion that mathematics is more important for boys than girls. In the advanced classes, three-fourths of the students had this opinion. It is interesting to note, however, that in both groups, about one student in five was undecided about this statement.

BOYS NEED MORE MATH THAN GIRLS

EIGHTH GRADE

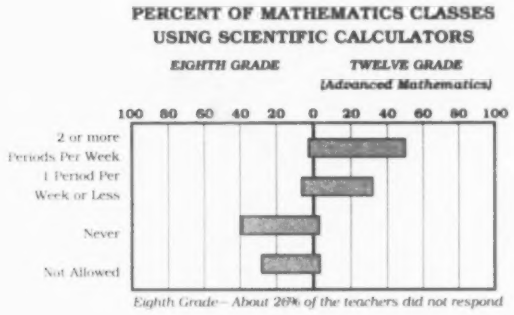


TWELFTH GRADE



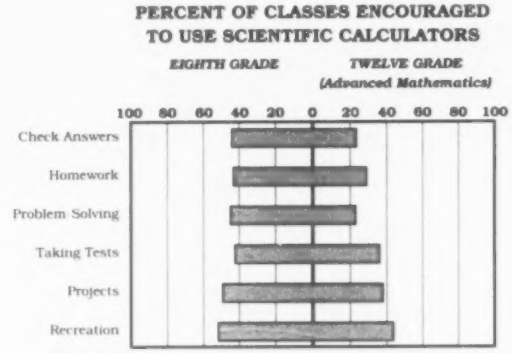
Calculator Use
Eighth and Twelfth Grade

The extent of calculator use in Virginia mathematics classrooms was found to be very limited at the eighth grade level, with very few classes reporting use more than one period per week and one-third of the classrooms reporting no use at all. Calculator use was much more frequent in Virginia's advanced mathematics classrooms.


PERCENT OF MATHEMATICS CLASSES
USING SCIENTIFIC CALCULATORS
EIGHTH GRADE
TWELVE GRADE
(Advanced Mathematics)
100 80 60 40 20 0 20 40 60 80 100
2 or more Periods Per Week
1 Period Per Week or Less
Never
Not Allowed
Eighth Grade– About 20% of the teachers did not respond

From the data reported in the previous section on "Student Attitudes" students at both levels see the calculator as a useful tool in mathematics.

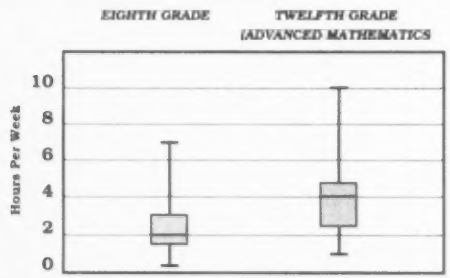
The kinds of calculator use in Virginia mathematics classrooms included checking answers and doing projects at the eighth grade level. Doing homework and problem-solving were more frequent uses reported at the advanced mathematics level. Recreational uses were rather common at both levels.



Homework Assigned
Eighth and Twelfth Grade

The amount of mathematics homework assigned in the United States is about two and one-half hours per week (30 minutes per day) at the eighth grade and almost twice as much (just over four hours per week) in the twelfth grade college preparatory mathematics classes. The Virginia data are presented here in "box-and-whisker" plots that show a similar pattern. The boxes include the middle fifty percent of the mathematics teachers reporting. The line across the middle of each box represents the median or middle score. So, the median amount of mathematics homework assigned to the advanced mathematics classes is close to four hours per week. That is, about fifty percent of the classes were reported as having less than four hours of homework assigned. The lines protruding from each end of the box (the "whiskers") encompass the middle 90% of the classes reporting. So, we see that 90% of the eighth grade classes were assigned between one-half hour and seven hours of mathematics homework per week

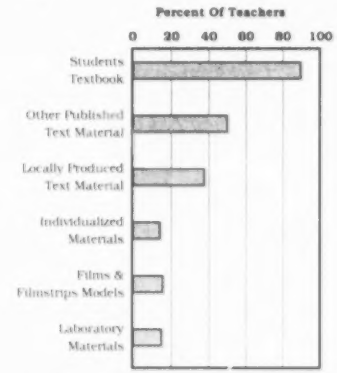
AMOUNT OF MATHEMATICS HOMEWORK ASSIGNED
VIRGINIA



With respect to mathematics homework, most eighth grade students reported spending one and a half hours per week on homework for the one mathematics class, while twelfth grade students spent from three to five hours per week for their one mathematics class.

Mathematics Instruction

Mathematics instruction in U. S. classrooms is clearly textbook-driven. The textbook largely determines what is taught as well as what strategies are used in teaching it. Even when not actually determining the content of instruction, the textbook almost always defines the boundaries for instruction. According to the reasons given by teachers for using various instructional strategies and representations of mathematical content, inclusion in the textbook did not always guarantee that a strategy or content representation would be used in the classroom. However, exclusion from the textbook made it virtually certain that the strategy or representation would not be used. Mathematics instruction in the U. S. seemed to have been mor e tied to the textbook than was instruction in most other countries. One could speculate as to reasons for this. For example, it could be because of the absence in many parts of the U. S. of syllabi and examinations that are beyond the local school or district level.



The student textbook is the predominant instructional resource in the U. S. eighth grade mathematics classrooms, and is so reported by ninety percent of the teachers. Other material, such as workbooks, and worksheets, were reported to be of secondary importance. Other materials, such as films and laboratory materials, seldom were cited as a primary resource, and were rarely or never used by the vast majority of teachers. This was true even for topics such as geometry and measurement were such materials might be considered most helpful.

How Students' Time Is Spent: Virginia

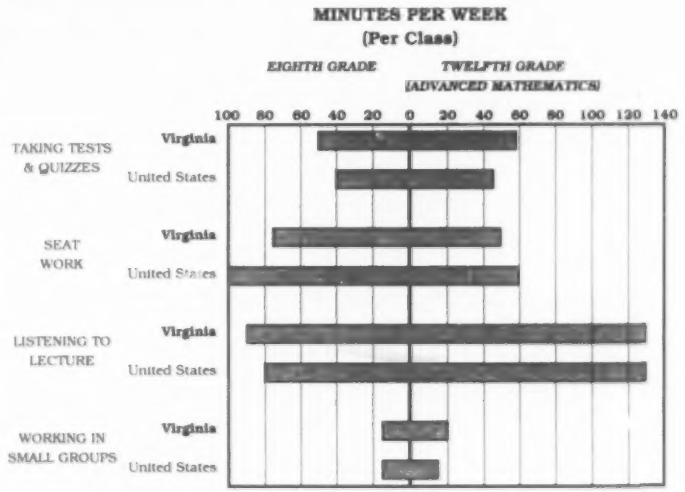
Teachers were also asked to estimate the average time spent by students in selected in-class activities. For eighth grade U. S. students the majority of time was devoted to individual work at their seats or to listening to lectures and explanations by the teacher. A little less than one class period per week was typically spent on taking tests and quizzes. Very little time was spent in work with small groups. The pattern was similar for twelfth grade students in the U. S., although they spent somewhat less time on individual seat work and relatively more time listening to teacher explanations.

How Students' Time Is Spent: Virginia
(Median Minutes Per Week)

	Eighth Grade	Twelfth Grade (Advanced Mathematics)
Taking Tests	50	48
Doing Seat Work	75	50
Whole Class Listening	90	130
Small Group	15	20

The VIMAP teacher report of how the students' time is spent, shows both eighth and twelfth grade students seem to be spending about 1 hour a week taking tests. The eighth graders are reported as having to do a little more seatwork, while twelfth graders do a lot more listening. Unfortunately, like the U.S., neither group spends very much time in small group activites.

The majority of student in-class time was thus seen to be devoted to one of two types of activities: listening to teacher talk or individual work with minimal interaction with the teacher. The two types of activities often seem to have occupied distinct phases of the class period. Little place seems to be given to guided, active discovery learning, in which students generated high-level questions and in which there was more of a balance between teacher and student subject-related talk. This pattern was not that different from that of other countries although the proportion of time spent on individual, in-class seat work was somewhat higher in the U. S.



The graph above shows how students' time in class was reportedly spent (in minutes per week for the classes sampled).

How Teachers' Time Is Spent: Virginia

One important aspect of VIMAP is to look at how teachers' time is spent. In Virginia, eighth grade teachers reported that they spent about 385 minutes (about 6 1/2 hours) per week doing certain instructional activities. Over 1 1/2 hours are spent on each of the following: planning, grading, and explaining new content. About 1 hour is spent on review weekly and only about 1/2 hour on administration and discipline. In the advanced mathematics classes considerably more time is spent planning and explaining; a little more time is spent grading papers and reviewing material; and much less time is spent on administrative and discipline matters.

Minutes Per Week (per class)

	Eighth Grade	Twelfth Grade (Advanced Mathematics)
Planning	100	142
Grading Papers	100	112
Explaining New Content	100	140
Reviewing Content	50	57
Routine Administration	25	15
Maintaining Order (Discipline)	10	3

Teachers' Use of Resources: Virginia (percent)

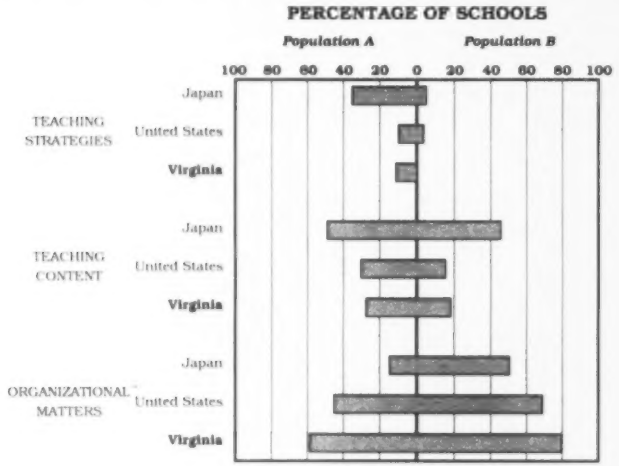
	Eighth Grade			Twelfth Grade (Advanced Mathematics)		
	R	S	O	R	S	O
Published Textbooks	7	44	47	2	4	94
Published Workbooks	27	44	16	49	40	11
Individualized Materials	25	47	25	78	19	3
Commercial Visual Materials	53	28	2	51	46	3
Commercial Tests	42	26	7	63	34	3
Self-written Textual Material	6	53	41	1	22	77
Self-written Tests	5	54	40	1	4	95

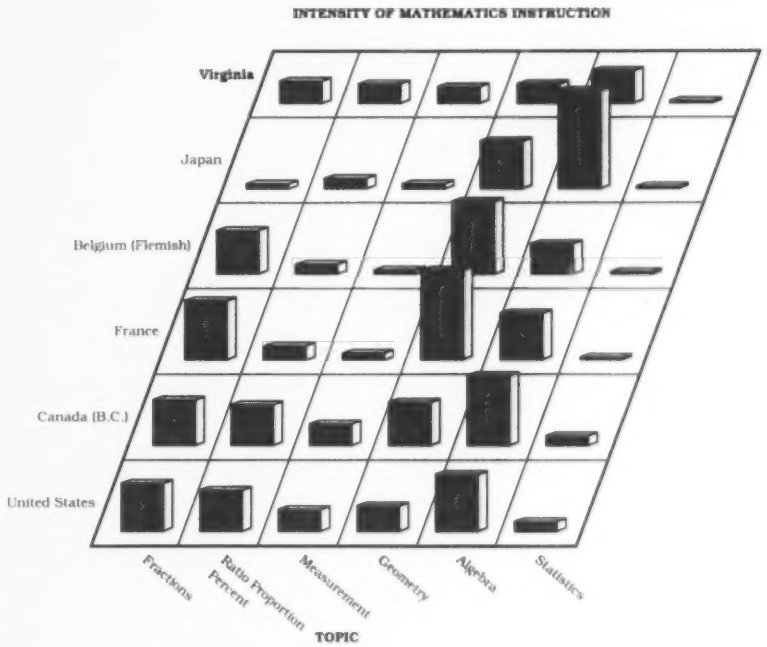
Note: R=rarely; S=sometimes; and O=often

Teachers in Virginia reported that the textbook was one of their major resources with over 90% of the eighth and twelfth grade teachers reporting regular use of the textbook. A large percent of Virginia teachers reported that they usually develop their own tests (99% of the twelfth grade teachers and 94% of the eighth grade teachers). Another heartening phenomenon in Virginia is the large percent (99% at the twelfth grade and 94% at the eighth grade) of teachers who rewrite and thereby adapt textual material for their own students.

Activities of Teachers

Activities at teachers meetings in the U. S. and Virginia at both grade levels tend to focus on organizational matters. Little attention was reportedly paid to teaching strategies or to content.

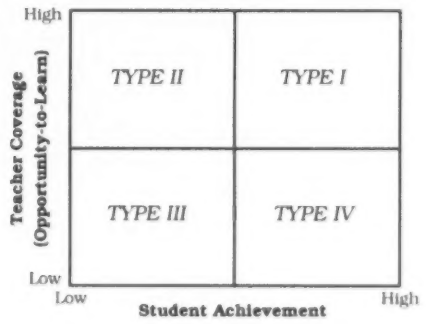




Intensity of mathematics instruction at the Population A level (eighth grade in the U. S.), based on teacher reports of anticipated percentage of class periods devoted to various topics, shows distinct patterns across countries. The Japanese curriculum provides their seventh graders with an intensive introduction to algebra. The Belgian and French programs focus on geometry and extensive work on common fractions (decimal fractions were dealt with in the elementary grades). In Virginia, as in the U. S., the curriculum has little intensity; but, a concerted effort across the entire curriculum.

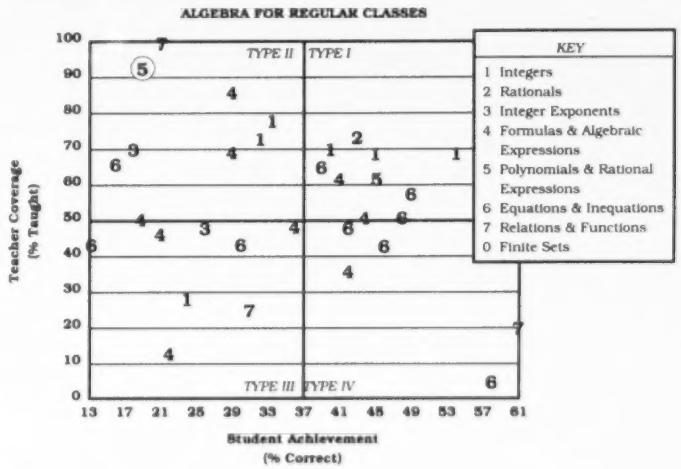
A unique feature of the Second International Mathematics Study is that it enables a detailed look at the teaching and learning of specific mathematical topics. One analysis that provides useful information is that of comparing reported teacher coverage of topics with student achievement on those topics.

The relation between teacher coverage and student achievement has already been demonstrated in this report. In fact, a major finding of the Second International Mathematics Study is that high student achievement is found in those countries where teacher coverage of those topics is high. With this in mind, it is of interest to look in some detail at the data from Virginia on teacher coverage and student achievement. We will classify the items from the international test in two ways: according to teacher coverage (high or low) and according to student achievement (high or low). As a result, each item can be one of four types, as the diagram below suggests:



Teacher coverage of the item is rated as "high" if it was taught to more than one-half of the classes. Similarly, student achievement on the item was rated as "high" if more than one-half of the students got the item correct.

The following graph demonstrates this analysis for one set of the Virginia data.



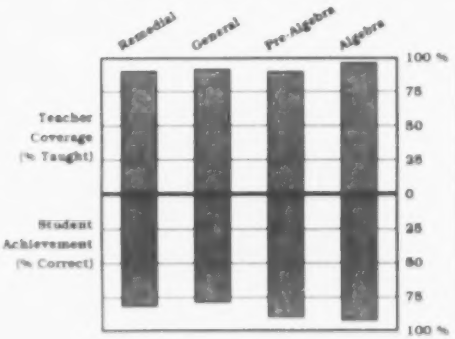
Each of the algebra items is represented by a single digit. The key identifies the mathematical content of the item. Note, for example, the circled "5" (Type II item). The item deals with polynomials and rational expressions. It was reported taught by most teachers (over 90%) but fewer than 20% of the students got it correct. For a complete analysis of each item see "Technical Report IV".

High Teacher Coverage and *High* Student Achievement (Type I Items)

These items perform as one would expect. A high proportion of teachers taught the content of the item and a high proportion of students got the item right.

The data are reported by the four eighth grade class types: Remedial, General, Pre-algebra and Algebra.

EIGHTH GRADE (ITEM A-11)



A team scored an average of 3 points per game over 5 games. How many points altogether were scored in the 5 games?

A $\frac{3}{5}$ D 5

B $\frac{5}{3}$ E 15

C 3

A-11

This item requires the student to know a considerable amount about the average (arithmetic mean) of a set of data--specifically, that the sum of a set of scores is equal to the average multiplied by the number of scores. As the graph indicates, teachers of all four class types reported high coverage of this topic (88% or more of the classes). And student achievement was high, with average scores of 78% or more in all class types.

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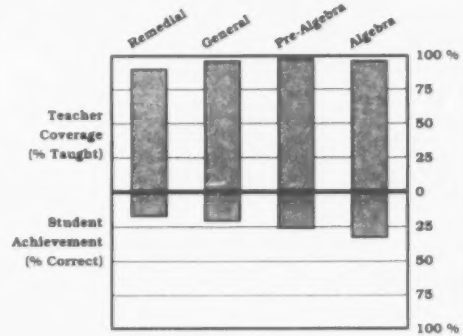
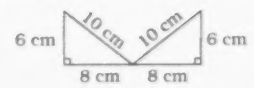
High Teacher Coverage and *Low* Student Achievement (Type II Items)

These items, if they deal with important curricular content, are of concern. Teachers give the content high coverage but students find it difficult.

EIGHTH GRADE (ITEM B-28)





The total area of the two triangles is?

A 6×8 cm^2

B $\frac{6 \times 8}{2}$ cm^2

C $\frac{10 \times 6}{2}$ cm^2

D $\frac{16 \times 12}{2}$ cm^2

B-28

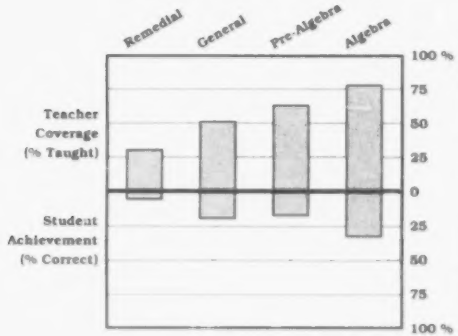
In view of the diffculty that students have with this item, perhaps this content is better left until later grades, when students have developed more in their ability to deal with a subject of this complexity. Alternatively, efforts could be made to find more effective ways to teach the content.

83

Low Teacher Coverage and *Low* Student Achievement (Type III Items)

These items, if they deal with important curricular content, are of concern for a different reason than for Type II items. Teachers are not teaching this content--and, correspondingly, students find it difficult.

EIGHTH GRADE (ITEM A-16)



x	3	6	P
y	7	Q	35

The table above shows the values of x and y, where x is proportional to y. What are the values of P and Q?

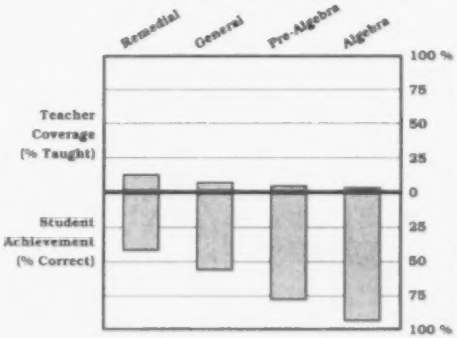
A P=14 & Q=31
B P=10 & Q=14
C P=10 & Q=31
D P=14 & Q=15
E P=15 & Q=14 A-16

As the data indicate, proportionality, as presented in a table format, is difficult for eighth graders--even for the algebra students. It could well be that professional development workshops could be profitably devoted to topics of proportionality--as it appears not only in tabular forms, but in varieties of problem contexts.

Low Teacher Coverage and *High* Student Achievement (Type IV Items)

These items are in some ways the most interesting. Teacher coverage is low but students do well on them.

EIGHTH GRADE (ITEM CORE 13)



If P=LW and if P=12 and L=3, then W is equal to?

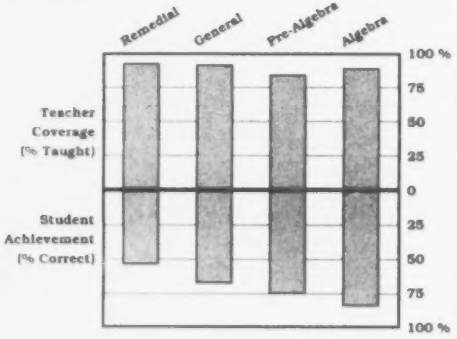
A $\frac{3}{4}$
B 3
C 4
D 12
E 36 Core 13

The content of this item, requiring some understanding of algebraic relationships, is by no means trivial. The achievement of students in every class type is encouraging. Perhaps this content is given some attention in the elementary grades.

With this background as to item types that were identified in the Virginia International Mathematics Assessment Project, further examples are provided.

Eighth Grade
***High* Teacher Coverage and *High* Student Achievement (Type I)**

Arithmetic



$\frac{1}{2} \times \frac{1}{4}$ is equal to?

A $\frac{1}{8}$

B $\frac{1}{6}$

C $\frac{2}{8}$

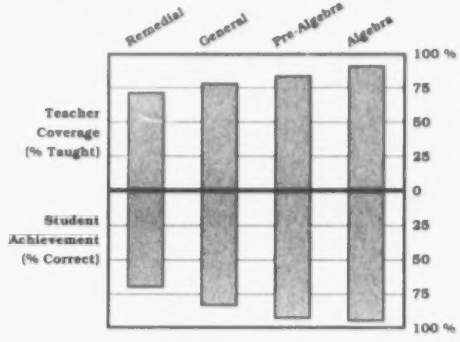
D $\frac{2}{4}$

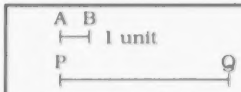
E 8

4-25

This routine arithmetic item received high coverage and was achieved well by most of the classes.

Measurement





The length of $\overline{AB}$ is 1 unit. Which is the best estimate for the length of $\overline{PQ}$?

A 2 units

B 6 units

C 10 units

D 14 units

E 18 units

0-8

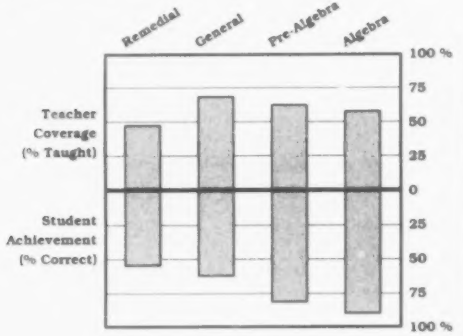
This estimation task, receiving high teacher coverage, was relatively easy for the students in all class types.

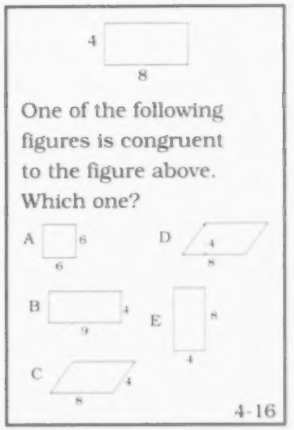
86

87

Eighth Grade
***High* Teacher Coverage and *High* Student Achievement (Type I)**

Geometry

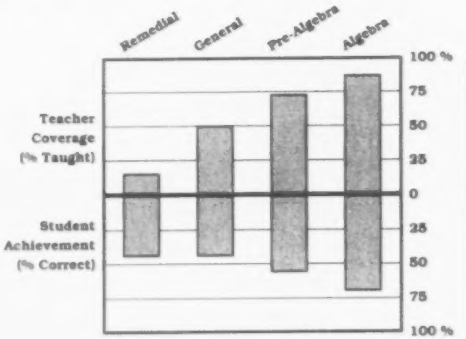




Student achievement corresponds to teacher coverage for this item testing congruence. About one-half of the remedial classes were taught the content and one-half of the remedial students were able to do the item correctly.

88

Algebra



The cost of printing greeting cards consists of a fixed charge of 100 cents and a charge of 6 cents for each card printed. Which of the following equations can be used to determine the cost of printing n cards?

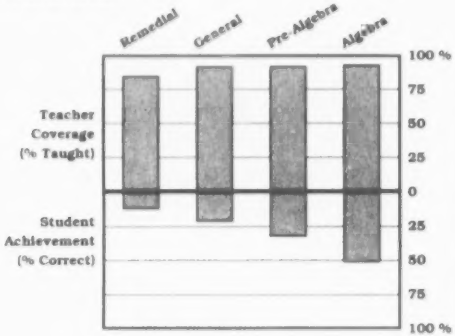
A cost=(100+6n) cents
B cost=(106+n) cents
C cost=(6+100n) cents
D cost=(106n) cents
E cost=(600n) cents 3-12

Formulating an algebraic expression from a word problem was learned by relatively large proportions of students--even in the remedial classes. Note, however, that this topic was taught by <u>very few</u> teachers of the remedial classes (that is, for the remedial classes, it is a Type IV item).

89

Eighth Grade
***High* Teacher Coverage and *Low* Student Achievement (Type II)**

Arithmetic



Four 1-liter bowls of ice cream were set out at a party. After the party, 1 bowl was empty, 2 were half full, and 1 was three quarters full. How many liters of ice cream had been EATEN?

A $3\frac{3}{4}$

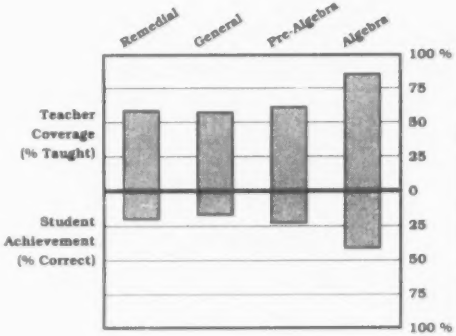
B $2\frac{3}{4}$

C $2\frac{1}{2}$

D $1\frac{3}{4}$

E None of the above

0-4



$150 is divided in the ratio of 2 to 3. The smaller of the two amounts is?

A $30

B $50

C $60

D $90

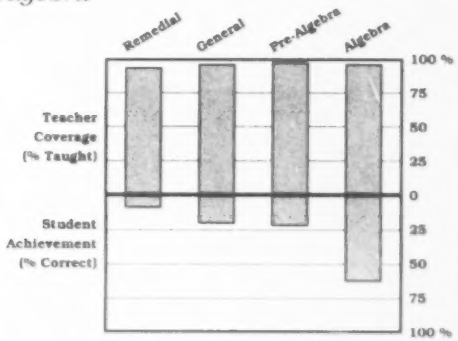
E $120

4-32

These were items that involved arithmetic operations (such as subtracting fractions or finding ratios) but were imbedded in a problem situation. Although students seemed to be able to perform routine arithmetic computations, they were less successful in identifying in a problem context those operations that needed to be performed.

Eighth Grade
***High* Teacher Coverage and *Low* Student Achievement (Type II)**

Algebra



m	-1	1	2	4
n	-1	3	5	9

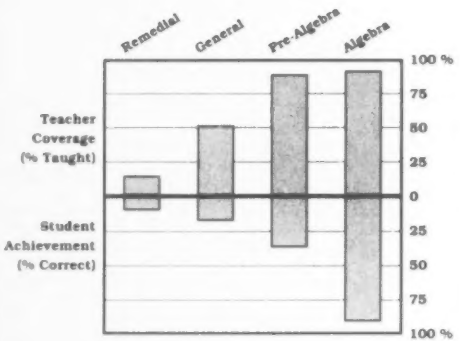
For the table shown, a formula that could relate m and n is?

A $n=m$ D $n=m^2+1$

B $n=3m$ E $n=2m+1$

C $n=-m^2+1$

2-5



Simplify:
$5x+3y+2x-4y$

A $7x+7y$

B $8x-2y$

C $6xy$

D $7x-y$

E $7x+y$

2-24

These items receive high coverage but are difficult for eighth grade students. Item 2-5, involving finding a rule to relate values in a table, received uniformly high coverage across the class types. Item 2-24, a standard item in simplifying algebraic expressions was difficult for all but the students in the algebra classes.

Note: Item 2-24 is more like a Type III item (it received low teacher coverage and was difficult for the students) for remedial students and a Type I item (it received high teacher coverage and was easy for the students) for eighth grade algebra classes.

92

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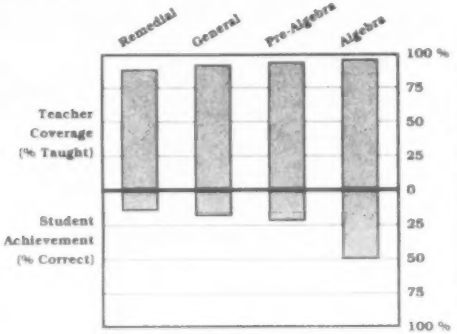
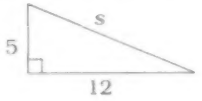
Eighth Grade
***High* Teacher Coverage and *Low* Student Achievement (Type II)**

Geometry

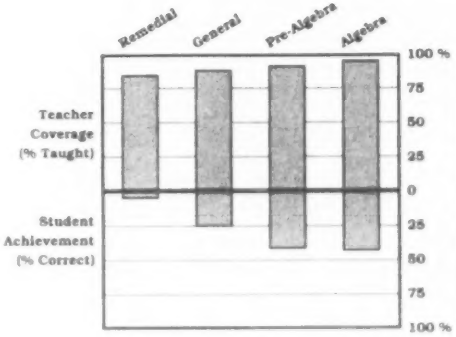




What is the value of s?

A 7
B 13
C 15
D 17
E None of these

3-1



If two triangles are SIMILAR, which of the following statements is TRUE?

A Their corresponding angles MUST be congruent.
B Their corresponding sides MUST be congruent.
C Their corresponding sides MUST be parallel.
D They MUST have the same area.
E They MUST have the same shape and size.

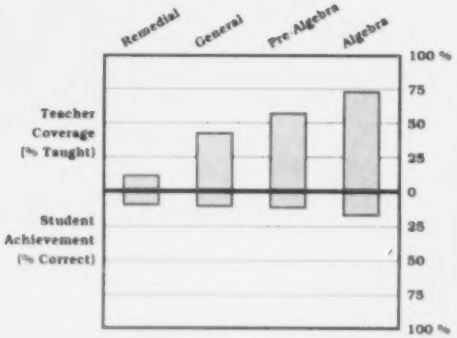
3-9

These items indicate that the Pythagorean Theorem and aspects of similarity are difficult for students.

Eighth Grade

***High* Teacher Coverage and *Low* Student Achievement (Type II)**

Measurement



A solid plastic cube with edges 1 centimeter long weighs 1 gram. How much will a solid cube of the same plastic weigh if each edge is 2 centimeters long?

A 8 grams
B 4 grams
C 3 grams
D 2 grams
E 1 gram

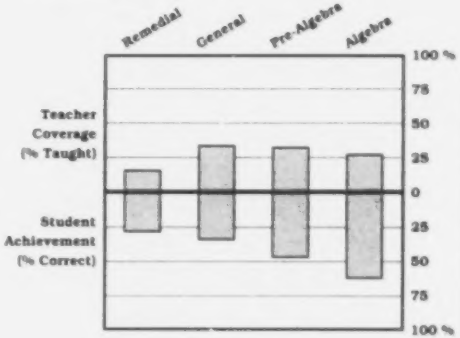
0-10

The problem of finding the volume of a cube whose edge is doubled is difficult, not only for students in Virginia, but was a very difficult item internationally.

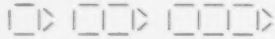
96

***Low* teacher coverage and *Low* student achievement (Type III Items)**

Arithmetic



Matchsticks are arranged as follows:



If the pattern is continued, how many matchsticks are used in making the 10th figure?

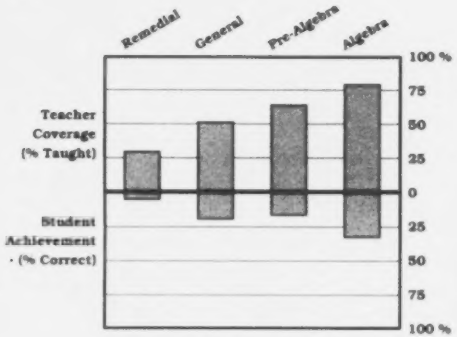
A 30
B 33
C 36
D 39
E 42

4-12

These tended to be non-routine items, such as finding patterns in numbers or identifying relationships in a table. These items require higher-level thinking skills.

Eighth Grade
***Low* teacher coverage and *Low* student achievement (Type III Items)**

Algebra



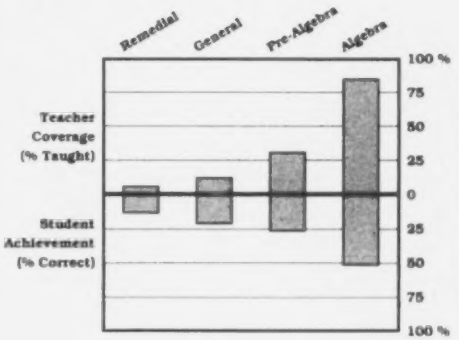
x	3	6	P
y	7	Q	35

The table above shows the values of x and y where x is proportional to y. What art the values of P and Q?

A P=14 & Q=31
B P=10 & Q=14
C P=10 & Q=31
D P=14 & Q=15
E P=15 & Q=14 1-16

This item may be classified as Type II or as Type III, it depends on the class type.

Algebra



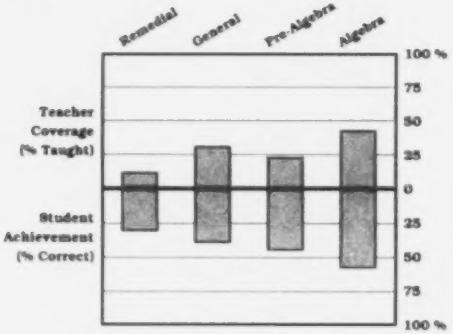
Soda costs a cents for each bottle, including the deposit, but there is a refund of b cents on each empty bottle. How much will Henry have to pay for x bottles if he brings back y empties?

A $ax+by$ cents
B $ax-by$ cents
C $(a-b)x$ cents
D $(a+x)-(b+y)$ cents
E None of these
4-27

This item was, generally, not taught, except to the algebra classes. Apparently, this content is not regarded as part of the eighth grade mathematics.

Eighth Grade
***Low* teacher coverage and *Low* student achievement (Type III Items)**

Statistics



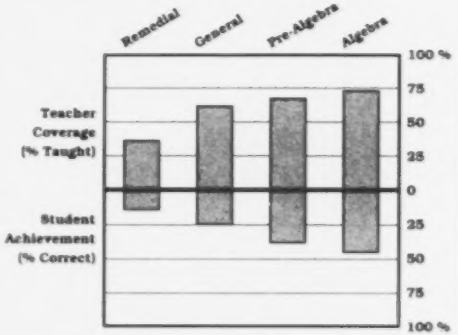
There are five black buttons and one red button in a jar. If you pull out one button at random, what is the probability that you will get the red button?

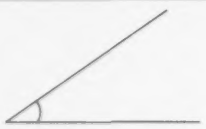
A 0
B $\frac{1}{6}$
C $\frac{1}{5}$
D $\frac{5}{6}$
E 1

3-6

Aspects of simple probability receive relatively little coverage by the eighth grade teachers. Student achievement is correspondingly low.

Measurement





The measure of the angle shown is nearest to?

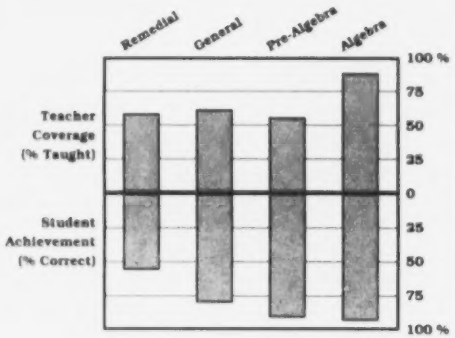
A 155
B 145
C 50
D 35
E 15

2-17

The topic of estimating the measure of an angle, as illustrated by this item, receives rather light teacher coverage.

Eighth Grade
***Low* teacher coverage and *High* student achievement (Type IV Items)**

Arithmetic



162 x 45 is equal to?

A 1378

B 1458

C 5890

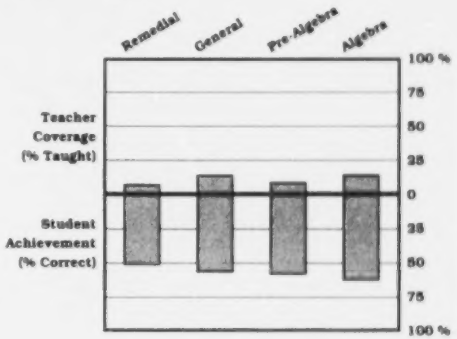
D 6290

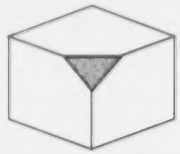
E 7290

3-28

Items of this type may be part of the curriculum of the elementary grades (that is, topics in grade school arithmetic with which the students are relatively proficient).

Geometry





The figure above shows a wooden cube with one corner cut off and shaded. Which of the following drawings shows how this cube would look when viewed from directly above? 1-15

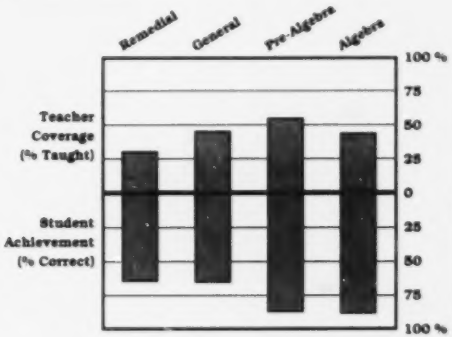
Spatial visualization, as represented by this item, was relatively easy for all students. Again, the performance of the remedial mathematics classes is noteworthy.

102

103

Eighth Grade
***Low* teacher coverage and *High* student achievement (Type IV Items)**

Statistics



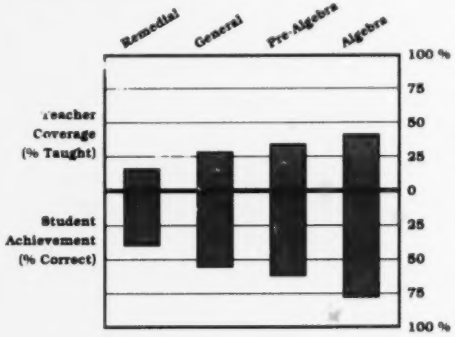
Joe had three test scores of 78, 76 and 74, while Mary had scores of 72, 82 and 74. How did Joe's average compare with Mary's?

A Joe's was 1 point higher
B Joe's was 1 point lower
C Both averages were the same
D Joe's was 2 points higher
E Joe's was 2 points lower

2-9

Work with averages, as represented by this item, received little teacher coverage. However the item was fairly easy for all of the eighth grade students.

Measurement



The area of the shaded circle is what part of the area of the larger circle?

A $\frac{1}{6}$
B $\frac{1}{5}$
C $\frac{1}{4}$
D $\frac{1}{3}$
E $\frac{1}{2}$

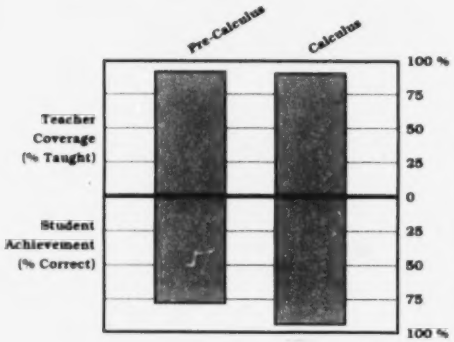
4-9

Teacher coverage of this estimation topic was light. However, student achievement was rather good.

Twelfth Grade
***High* teacher coverage and *High* student achievement (Type I)**

Advanced Algebra

Topic: finding solution sets



The solution set for the equation $(1-2x)(2+x)=0$ is?

A $\{\frac{1}{2},-2\}$

B $\{-\frac{1}{2},2\}$

C $\{-1,-2\}$

D $\{\frac{1}{2},-2\}$

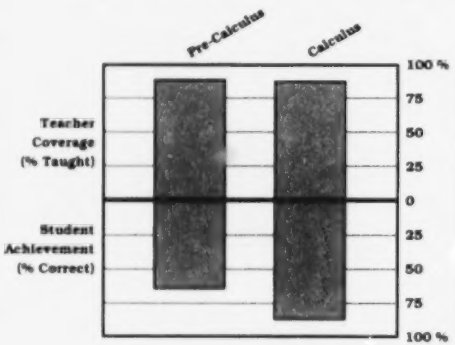
E $\{2,-2\}$

2-14

166

Elementary functions

Topic: properties of functions



If $xy=1$ and x is greater than 0, which of the following statements is true?

A When x is greater than 1, y is negative.
B When x is greater than 1, y is greater than 1.
C When x is less than 1, y is less than 1.
D As x increases, y increases.
E As x increases, y decreases

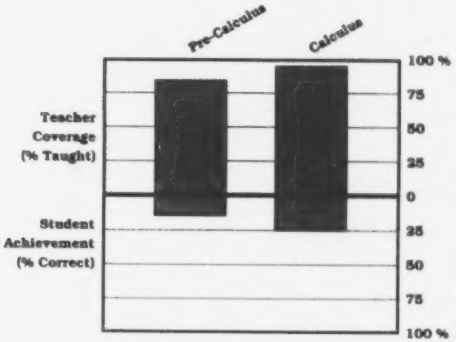
1-10

167

Twelfth Grade
***High* teacher coverage and *Low* student achievement (Type II)**

Advanced Algebra

Topic: trigonometric relations



An angle θ is known to be between 90 and 180 and $\cos^2 \theta = \frac{16}{25}$.

The value of sin 20 is then?

A $-\frac{24}{25}$ D $\frac{7}{25}$

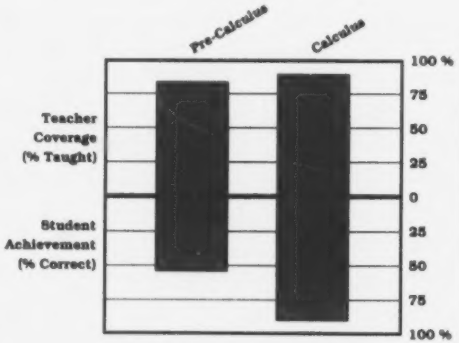
B $-\frac{15}{25}$ E $\frac{24}{25}$

C $-\frac{7}{25}$

4-7

Elementary functions

Topic: composition of functions



The functions f and g are defined by $f(x) = x - 1$ and $g(x) = (x + 3)^2$.

$g(f(x))$ is equal to?

A $(x-1)(x+3)^2$
B $(x+3)^2 - 1$
C $(2x-2)^2$
D $(x+2)^2$
E x^2+8

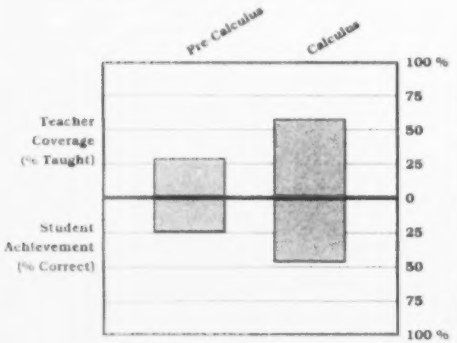
8-12

Note: For the calculus classes, this was a high coverage - high achievement (Type I) item.

Twelfth Grade
***Low* teacher coverage and *Low* student achievement (Type III)**

Advanced Algebra

Topic: parametric equations



What is the equation in x and y of the curve with parametric equations

$x = t + \frac{1}{t}, y = t - \frac{1}{t}$?

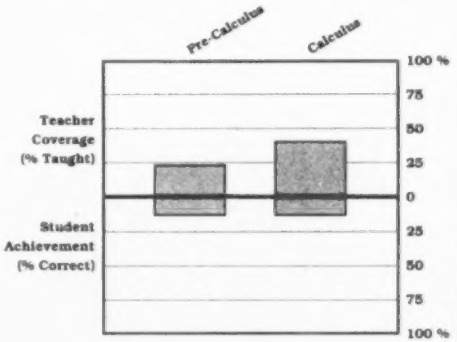
A $x + y = 1$
B $x + y = 2$
C $x^2 + y^2 = 4$
D $x^2 - y^2 = 4$
E $2x^2 - y^2 = 4$

6-8

110

Elementary functions

Topic: logarithmic functions



Let $y = 4x^3$, with x and y taking positive real values. When $\log x$ is plotted against $\log y$ the resulting graph will be?

A a single point
B a cubic curve
C a parabola
D a straight line
E an exponential curve

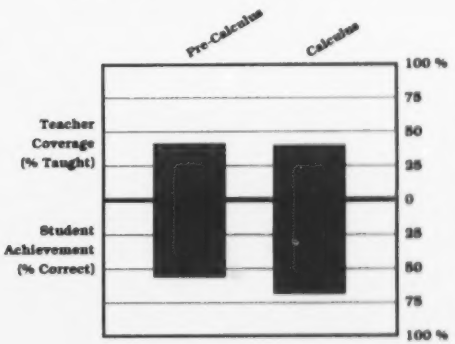
8-17

111

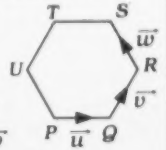
Twelfth Grade
***Low* teacher coverage and *High* student achievement (Type IV)**

Advanced Algebra

Topic: vector operations



PQRSTU is a regular hexagon in which $\overrightarrow{PQ}$ represents vector $\vec{u}$, $\overrightarrow{QR}$ represents vector $\vec{v}$, and $\overrightarrow{RS}$ represents vector $\vec{w}$. $\overrightarrow{PT}$ represents?



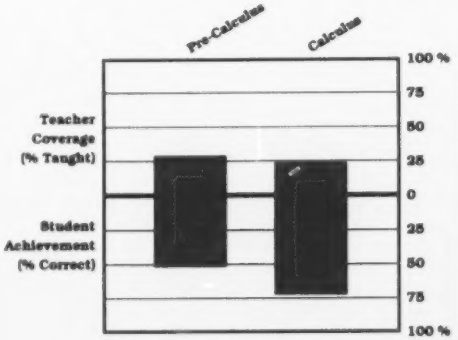
A $2\vec{u} + \vec{v}$
B $\vec{u} + \vec{v} + \vec{w}$
C $2(\vec{u} + \vec{v} + \vec{w})$
D $2\vec{u} + \vec{v} + \vec{w}$
E $\vec{v} + \vec{w}$

7-15

112

Probability

Topic: elementary theoretical probability



A set of 24 cards is numbered with the positive integers from 1 to 24. If the cards are shuffled and if only one is selected at random, what is the probability that the number on the card is divisible by 4 or 6?

A $\frac{1}{6}$
B $\frac{5}{24}$
C $\frac{1}{4}$
D $\frac{1}{3}$
E $\frac{5}{12}$

4-14

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Rationale for the Targeted Study

The purpose of the Virginia International Mathematics Assessment Project was to develop a model for program assessment and evaluation at the state and local school levels in order to provide detailed high quality data that would be of use to curriculum supervisors, classroom teachers, and others in assessing the current curriculum and in identifying new directions for improvement. Hence, in order to look at the usefulness of the SIMS model at the local level, a study of the mathematics programs in two selected school divisions was conducted.

Target Populations

The target populations for the local school divisions were the same as those for the state study. This included all students enrolled in mathematics classes at the eighth grade level (Population A) and students enrolled in mathematics classes at the twelfth grade level having a prerequisite of two years of algebra and one year of geometry (Population B).

What Did the Targeted Study Involve?

All classes in the target populations for both school divisions were tested. There was one significant difference between the study at the local level and the study at the state level.

A pre-test and post-test were administered. This provided data on student achievement both at the beginning and at the end of the school year. The Pre-test data provided baseline data for teachers to use during the school year.

Orientation for School Personnel. An orientation meeting was held for administrators and teachers of the mathematics classes selected for the study. Participants were briefed on the purpose of VIMAP and the role the targeted school divisions would serve in the assessment project. Also, the curriculum analysis, which assessed the adequacy of the international test items for use in Virginia, was discussed.

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During the meeting, teachers were told which of their classes would be involved in the project and given instructions on what their responsibilities would be. Teachers were given individually prepared student packets of materials for each of their classes. Student packets contained test forms, answer sheets, and a student questionnaire. Teacher questionnaires and other materials were also distributed at the meeting.

Testing. Students in the targeted classes were tested soon after the beginning of the school year. The eighth graders answered the Core Form test and one of four randomly assigned rotated forms; the twelfth graders were randomly assigned two of eight rotated test forms. Students were asked to fill out the student questionnaire providing valuable background information for curriculum and instructional use.

Interpretation and Use of Pre-test Results. The State's Mathematics staff met with the participating teachers in each school division separately to assist them in interpreting pre-test results. Teachers were given the achievement results of each of their classes by test form and by item. Summary data for the Population A schools and for the Population B schools were discussed and suggestions were made on how the data might be used.

Teachers analyzed the data in light of what mathematics they had taught prior to pre-testing and the content yet to be covered in the local curriculum. The analysis of data was used to focus instruction on content the teachers considered important and where achievement was low.

Post-testing. The targeted mathematics classes were post-tested near the end of the school year. The same test forms taken in the beginning of the year were used. This made it possible to evaluate the achievement during the school year. Also, data from the post-test could be compared with state and international estimates.

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Analysis of Eighth Grade Data

The pre-test and post-test achievement results were reported for each class and for all eighth grade classes in the school divisions. Achievement on items appearing on the Core Form was reported by groups of students taking each of the four rotated forms. The tables below illustrate the types of information reported.

The analysis of achievement included a summary of the test statistics for all items on a given form. An example, of a pre-test summary report appears in the table below. Individual class reports give the same types of information.

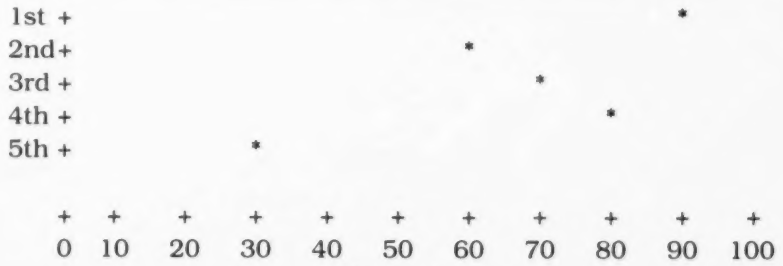
Pre-Test Summary of Statistics on Core Form Items

Number of items	40	Possible low score	0
Mean score	14.30	Possible high score	40
Median score	14.86	Obtained low score	0
Standard Deviation	-	Obtained high score	13
Reliability (KR-20)	-	Number of scores	98
S.E. of Measurement	-	Blank scores	0
		Invalid scores	0
		Valid scores	98

The analysis of achievement also included information on each test form by item. Five quintiles of achievement scores were established for overall student achievement on each form. The number and percent of students in each quintile who answered the item correctly was reported. Also, the percent of students overall who answered the item correctly and each of the distractors is given.

Percent of Correct Responses by Fifths

Quintiles



Statistics in the graph above were taken from the following item.

Item 1: 2 meters + 3 millimeters is equal to

A. 2.0003 meters
*B. 2.003 meters
C. 2.03 meters
D. 2.3 meters
E. 5 meters

*B is the correct choice

The number of students in each quintile and the percent of students overall who chose the correct answers and each distractor is given in the chart below.

Matrix of Responses By Fifths

Quintiles Answer Choices	A	(B)	C	D	E	Omit
			Number of Students			
1st	0	17	0	0	0	2
2nd	1	10	0	0	0	5
3rd	0	16	0	0	0	6
4th	0	17	0	0	0	5
5th	0	7	0	0	0	12
Proportion of Students Answering						
	.01	(0.68)	0	0	0	0. 31
Discriminating Factor						
	.06	(0.32)	0	0	0	- 0. 34

The three preceeding tables illustrate the types of information reported in the item analysis using a specific test item from the Core Form. The data is a sample from a pre-test summary report. Individual class reports give the same types of information.

Analysis of Twelfth Grade Data

The pre- and post-test achievement results were reported by test form and by item for each class and for all twelfth grade classes in the targeted study. Achievement data was reported in the same way as the eighth grade population.

Comments on Implementation at the Local Level

The implementation was successfully carried out with little disruption to the schools involved. The evaluation process provided the impetus for teachers and school administrators to examine their mathematics curriculum and to make sound educational decisions about changes needed. The two school divisions were also able to compare the achievement results of their students to both the Virginia and SIMS data.

The experience caused teachers to reflect on instructional practices and to question the importance of some topics that are currently taught. The opportunity to examine an international mathematics curriculum provided an inservice in and of itself.

The data provided information that will assist the local schools in monitoring and assessing their curricula, their in-service activities, and their student performance from a state and an international perspective.

Eighth Grade Findings

1. Overall, the mathematics achievement of eighth grade students in Virginia public schools exceeds that of eighth grade mathematics students across the United States. In the content areas of arithmetic, algebra, and statistics, Virginia students scored higher than the U. S. average; in geometry, they are identical; and in measurement, Virginia students scored only one point lower than the U. S. average. In addition, Virginia students scored higher than the international average in three of the five topic areas (arithmetic, algebra and statistics).

2. In Virginia as in the United States as a whole, there is not just a single eighth grade mathematics curriculum. Instead, there is the typical country-wide pattern of four distinct eighth grade mathematics programs: Remedial (for low-achieving students); General (for average students); Pre-algebra (for above-average students); and Algebra (for accelerated students).

3. The four programs differ greatly in terms of the kind and extent of mathematics content. The Remedial students are provided, mainly, with topics from arithmetic, with little exposure to geometry or algebra. The General students are offered a review of topics in arithmetic and some exposure to topics in geometry, algebra and statistics. The Pre-algebra students are provided more content in algebra and geometry than are the General students. The Algebra students are offered, typically, a freshman algebra course. These algebra students are, however, offered little geometry, probability, or statistics.

4. As a general pattern in Virginia (as in the United States as a whole), the level of achievement of eighth grade mathematics students corresponds to the amount of mathematics that they are taught. For example, in algebra, the students in the Remedial classes have the lowest achievement scores (they are taught very little algebra), and the students in the Algebra classes have the highest scores (they are taught a relatively large amount of algebra).

5. Some mathematics topics are relatively easy, on average, for students in all four eighth grade mathematics programs, even if the topics have not been taught at the eighth grade level. For example, spatial visualization was reported as not having much coverage in any of the four eighth grade programs. However, student achievement on items dealing with this topic was rather good (and at about the same level) in all four programs.

6. Other mathematics topics (for example, the Pythagorean Theorem) appear to be relatively difficult for most eighth grade students, even though the topic receives high teacher coverage. Such topics, if considered important for the eighth grade curriculum, would appear to need more effective instructional approaches (including, perhaps, the use of technology).

7. Eighth grade students tend to have positive attitudes about mathematics. They want to do well in mathematics and would like to work at a job that uses mathematics.

SUMMARY

Twelfth Grade Findings

1. The achievement of twelfth grade advanced mathematics students in Virginia is, on average, a few points higher than that of their counterparts across the United States. Especially noteworthy is the higher Virginia achievement in algebra and calculus.

2. In most countries, (according to the SIMS report) all advanced mathematics students (twelfth grade) take calculus. Virginia, however, is like the United States as a whole in that only about one-third of the advanced students are enrolled in calculus classes.

3. The achievement of the calculus students in Virginia is relatively high--about 8 percentage points above their U. S. counterparts and 11 points above the international average. The achievement of the twelfth grade pre-calculus students was, like that of pre-calculus students across the United States, rather low. For example, performance was weak on many aspects of routine algebra content.

4. Some mathematics topics, for example, exploring basic properties of a function, received high teacher coverage and tend to be easy for advanced mathematics students. Other topics, such as elementary probability theory, are found easy by students even though they tend not to be covered by teachers of the target classes. Still other topics are difficult, even though they receive high teacher coverage--for example, dealing with certain trigonometric relationships.

5. The attitudes of the twelfth grade students toward mathematics tend to be positive. They believe that knowing mathematics will help them get a good job and that their parents want them to do well in the subject.

...the central problem for today and tomorrow is no longer access to school. It is access to knowledge for all. The true challenge is that of assuring both equity and quality in school programs.

J. Goodlad

(1985b)

Achieving Excellence in Mathematics in Virginia's Schools: Some Recommended Next Steps

1. Upgrading the content of the curriculum

In Virginia, the eighth grade mathematics that is available to most students includes repeated visits to grade school arithmetic. As the SIMS report for the United States has observed, from an international point of view, eighth grade mathematics in the U. S. resembles much more the end of elementary school than the beginning of secondary school (The Underachieving Curriculum, page 95). The content of the curriculum should therefore be upgraded to reflect recent developments in mathematics and the demands of society and post-secondary education for quantitatively literate high school graduates. Data from this report, the Virginia SOL for Mathematics and national efforts such as the report of the National Research Council (1989), Everybody Counts, provide useful guidelines for such curriculum reform activity.

2. Restructuring the eighth grade mathematics curriculum

In Virginia, as in the United States, there is not an eighth grade curriculum. Instead, there are four programs that differ greatly in the amount of mathematics made available to students. The lower-ability students, in particular, are provided mainly a repetition of topics in arithmetic from the grade school curriculum. The eighth grade curriculum should be restructured so that all students have the opportunity to learn something of the power and utility of mathematics. The Virginia International Study has demonstrated, that as students' opportunity to learn topics in mathematics increases, their achievement tends to increase accordingly. Again, data from VIMAP, SOL, and certain national studies provide important guidelines as to what should be included in this common core of mathematical knowledge, not only for eighth grade, but for each grade K-12.

3. Increasing the number of students who study mathematics: A PUMP, NOT A FILTER

Everybody Counts the report of the National Research Council (1989), has observed that, "more than any other subject, mathematics filters students out of programs leading to scientific and professional careers... on average, we lose half the students from mathematics each year (from ninth grade through graduate school)..." (page 7). In Virginia, we need to find ways to retain more students in academic mathematics courses through high school. This can happen as the result of a variety of factors. For example, recommendation 1 can, among other things, serve to make mathematics a more attractive subject to study. Recommendation 2 will provide more youngsters with a better view as to what mathematics as a field of study is all about. Furthermore, if we seek to increase enrollments in mathematics courses, we should capitalize on the positive attitudes of Virginia students toward mathematics. As noted in our report, large proportions of our students believe that it is important to know mathematics in order to get a good job. Students overwhelmingly feel that their parents want them to succeed in mathematics.

4. Clear expectations for achievement

As new instructional programs are put into place, it will be important to clarify for students the expectations for achievement that go along with these programs. An important part of this implementation phase will be effective methods for assessing how much mathematics students learn, and how they learn it. We need mathematics assessment that matches the state's curriculum and focuses on what students know and can do, rather than what they do not know. The NCTM Curriculum and

Evaluation Standards for School Mathematics provides useful guidelines as assessment procedures are established at the classroom, school, and division levels.

5. Classrooms as places for the teaching and learning of mathematics

In Virginia, as in the United States as a whole, mathematics classrooms tend to be textbook-dominated. In many cases, relatively little use is made of instructional resources such as video-tapes, mathematical models or manipulatives. Instructional use of computers and calculators as reported by teachers is rather limited as well. Ways should be found to enrich the climate of the mathematics classroom, providing for every youngster the experience of encountering mathematical ideas in a supportive, exciting, and productive environment.

Epilogue

In the early 1980s, the National Science Board prepared for the National Science Foundation and the American people a report that outlined a plan of action for "improving mathematics, science and technology education for all American elementary and secondary students so that their achievement is the best in the world by 1995" (Educating Americans for the 21st Century, 1983). In the intervening years since the publication of that report, many initiatives at the national, state, and local levels have been undertaken to help accomplish the ambitious goals of the National Science Board. The Virginia International Mathematics Assessment Project is an important part of the ongoing commitment we have to help assure that our students will continue to benefit from the vision of excellence of the National Science Board and of our state Department of Education.

"Our mission is to prepare students for life in the 21st century. We must empower our young people with the skills and knowledge necessary to succeed in a world that has not yet been defined."

Governor Elect L. Douglas Wilder

List of Technical Reports

Over and beyond the Main Report for the Virginia International Mathematics Assessment Project, there are five technical reports. A brief description of each report is listed below. These reports are available on a limited basis from the Commonwealth of Virginia; Department of Education; Mathematics Service; P.O. Box 6Q; Richmond, Virginia 23216-2060.

I. **Virginia International Mathematics Assessment Project: Mathematics Counts in Virginia "Technical Report I: Data Summary"**

This report includes a summary of data from all questionnaires and all instruments used in the Study to collect data. This includes information from the Student Questionnaire Population A and Population B, Teacher Questionnaire Population A and Population B, Teacher General Classroom Processes Questionnaire Population A and Population B, School Questionnaire Population A and Population B and Opportunity to Learn and Achievement Questionnaires for Population A and Population B.

II. **Virginia International Mathematics Assessment Project: Mathematics Counts in Virginia "Technical Report II: Test Instruments"**

This report includes a copy of each test booklet used in the study for both Population A (eighth grade) and Population B (twelfth grade).

III. **Virginia International Mathematics Assessment Project: Mathematics Counts in Virginia "Technical Report III: A Guide for Replicating the Second International Mathematics Study"**

This report is intended to serve as a guide for schools, school divisions and/or states for replicating the Second International Mathematics Study. It includes detailed information on the steps to follow in carrying out a study.

IV. **Virginia International Mathematics Assessment Project: Mathematics Counts in Virginia "Technical Report IV: Teachers Teaching...Students Learning"**

This report includes a classification of each test item used in the Virginia International Mathematics Assessment Project according to teacher coverage and to student achievement. As a result, the information therein allows the reader to take a detailed look at the teaching and learning of specific mathematical topics.

V. **Virginia International Mathematics Assessment Project: Mathematics Counts in Virginia "Technical Report V: Sampling"**

This report includes a description of the Sample Design for the Virginia International Mathematics Study for each of the two populations, Population A and Population B. The report also includes information that may be helpful to someone planning to replicate SIMS, such as: purpose of the sampling activities, procedures used to draw samples, quality control procedures and other such information.

APPENDIX B

Design of Samples for Statewide Mathematics Assessment in Virginia

Target Populations

The target populations for the study are:

Population A: Students enrolled in mathematics classes offered at the eighth grade level.

Population B: Students enrolled in mathematics classes at the twelfth grade level (classes requiring a prerequisite of two years of algebra and one year of geometry).

The study is limited to regular public schools in Virginia. Private schools are not included. According to Virginia Department of Education data, in the 1985-1986 school year, public schools had 95 percent of enrollment at the 8th grade level in public and private schools in Virginia, and 95 percent of enrollment at the 12th grade level.

Sample Design -- General

The sample for each of the two student populations was a three-stage probability sample of students designed to be approximately self-weighting; i.e., to give each student in target population approximately the same probability of selection for assessment. The sample of schools for students of 12th grade mathematics were selected independently of the sample of schools for students of 8th grade mathematics. Eligible mathematics classes in schools selected in the sample were the second-stage sampling units.

At each grade level there are sets of test items in separate booklets rotated among students in the study sample, paralleling the design used in the Second International Mathematics Study. The study design specified that all students in the sampled classes be tested. Within a class the booklets were spiralled (rotated) so that a fraction of the students responded to any item that appears in only one of the booklets (i.e., a spiralled item). This constituted the third stage of sampling. The design at each grade level differed somewhat in the following way. In the eighth grade, four spiralled (rotated) forms were used, together with a 40 item test (core test) taken by all students. In the twelfth grade, eight rotated forms were used; a core test was not used.

The number of schools and students to be selected for each of the populations was determined, on the basis of available data on components of variance in the U.S. SIMS study, to meet sampling reliability objectives for the assessment. These objectives were to provide estimates of the percent of 8th grade students who would correctly answer a given test item, with a standard error of 5 percentage points at the ninety-five percent level of confidence, and similarly for 12th grade students. It was also an objective to provide estimates of the average percent of items correctly answered by 8th grade students and 12th grade students. The standard errors for these estimates would be expected to be smaller than the standard error for individual items.

Sample Design -- 8th Grade Level

The sample for students of 8th grade mathematics was based on a stratified sample of 52 schools. Twenty-six strata of eligible schools were established, each representing approximately the same number of 8th grade mathematics classes, and a sample of two schools was randomly selected per stratum.

For purposes of stratifying schools, the average 8th grade SRA mathematics score for each School Division for the previous school year and enrollment by school was available centrally.

School Divisions having 8th grade enrollment of 500 students or more were asked to provide the average 8th grade SRA mathematics score for each school in the Division. These divisions accounted for approximately seventy-five percent of 8th grade enrollment. In the stratum of large divisions schools were stratified by their average SRA mathematics scores, and in the stratum of Divisions with 8th grade enrollment under 500, schools were stratified by division average 8th grade SRA mathematics score. Four strata of SRA scores were defined. Within each of these strata, schools were further stratified by enrollment size and sampling strata defined so as to represent approximately an equal number of 8th grade classes. A sample of two schools per stratum was then selected with equal probability, and an expected four classes subsampled per selected school. (Schools estimated to have fewer than four 8th grade classes were sampled at the overall rate and all classes in the selected schools were included in the study.)

For sampling classes within schools, the selected schools, were asked to provide a list of their 8th grade mathematics classes with some indication as to ability grouping, if any, by class within school. Small classes were grouped as feasible to provide sampling units of approximately the same size. The sampling units were then randomly selected. In large schools the classes were further grouped into two strata within the school prior to sampling and two sampling units were selected per stratum.

Sample Design -- 12th Grade Level

The design for a sample of students in 12th grade level mathematics classes (Population B) paralleled that for the sample of 8th grade students.

Twenty-six strata of eligible schools were established, and schools were selected by simple random sampling per stratum. Schools were stratified by two criteria: (1) average 11th grade SRA mathematics score for the school based on the most recent testing or, if not available, the average for the Division to which the school belongs; and (2) a measure of the size of enrollment in the 12th grade level mathematics. Four strata of SRA scores were defined, and within these, schools were further stratified by mathematics enrollment size. Average score by school was available for schools having about 90 percent of the eligible enrollment. The measure of enrollment size (MOS) used for the size stratification of individual eligible schools was based on centrally available data by schools as to enrollment in the 12th grade level mathematics classes (E), as follows:

E	MOS
under 100	1
100-199	E/23.1 rounded up
200 and over	E/25.1 rounded up

Strata were defined to be of approximately equal size in terms of the MOS. A sample of two schools per stratum was selected with equal probability and 12th grade level mathematics classes were subsampled to provide an approximately self-weighting sample with an expected 100 eligible students tested per school. There are 194 schools with estimated 12th grade enrollment under 100. These were sampled at the overall student rate and all students in the selected schools were tested.

There were 8 test booklets but two testing sessions per student so that, again, any spiralled test item was taken by a fourth of the students.

Summary of the Sampling and Cooperation Rate

The final results for the sampling and testing are as follows:

	Population A (regular 8th grade mathematics classes)	Population B (12th grade level mathematics classes)
Schools		
Number in sample	52	52
Number cooperating	51	51
Cooperation rate	98 %	98 %
Eligible mathematics classes		
Number in sample	216	169
Number not approached	2	-
Net	209	169
Number tested	209	169
Cooperation rate	100 %	100 %
Eligible students		
Enrollment in tested classes	4,612	3,500
Number tested (records in the assessment file)	4,506	3,331
Cooperation rate	95 %	95 %